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COMPARATIVE ANALYSIS OF LIVE LOAD DISTRIBUTIONON EXTERIOR RIGHT BRIDGE DECK GIRDERS UNDER LOAD MODEL 1 (EN 1991-2) USING DIFFERENT METHODS

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Abstract
In this paper, three models of slab on beam (T-beam) bridge with varying number of girders and varying spans lengths were loaded with Load Model 1 (LM1) according to Euro code 1 Part 2 (EN 1991-2), and analysed using Finite Element Analysis, Grillage Analogy, and Courbon's method. The width of the carriageway is 7.2m, and with Euro code specifications, two notional lanes (3m wide each) and a remaining area 1.2m wide was produced for all the models. The bridge was analysed for LM1 only, with the load arrangement maximised for worst effect on the exterior girder. In the results obtained, finite element analysis gave the most economical results for longitudinal bending moment and shear forces, followed closely by grillage analogy. However, a calibration factor was proposed for the results from Courbon's method as a function of the bridge span length, which will enable Courbon's method to be used as a quick check for verification of results from computer methods, since it is a very easy and quick manual method to apply, thereby ameliorating the limitations in the use of the method.

Keywords: T-beam Bridge, Load Model 1, Staad Pro, Finite Element Analysis, Grillage Analysis, Courbon's Method, Calibration factor

Introduction
A bridge is a structure designed to span over obstacles, and hence, they play important roles in the development of a city both in terms of transportation, aesthetics, and otherwise. The design of bridges has evolved over the years and a lot of solutions are considered when selecting the choice of bridge deck to adopt. One of the many types of bridge decks is the T-beam and slab system, where the slab is usually cast in-situ over precast beams. Slab and beam decks are economical for short and medium span bridges between 10m to 25m (Grandic et al, 2014, Praful and Hanumat, 2015). For beam and slab deck bridges, there may be transverse cross girders, while in many bridges the transverse beams in the span are omitted for reasons of simplicity of bridge deck construction. Bridge decks with no transverse beam(s) in the span have less effective but still important transverse distribution achieved by transverse beams over the supports and deck slab. In this case the torsional stiffness of main structural elements (longitudinal and transverse beams and deck slab) has a great influence on transverse load distribution (Yousif and Hindi, 2007). This is the commonest bridge deck concept in Nigeria.

EN 1990 Annex A2 and EN 1991 Part 2 covers the design of road, rail, and foot bridges in Europe. Bridges are expected to be able to carry all permanent, variable, and accidental actions

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that it may be subjected to in its design life. One of the major variable actions that bridges are subjected to is the load from traffic. It is pertinent to note that while traditional bridge codes used real vehicles for static loads, modern codes such as the Euro code replaced real traffic loads with artificial load models for static verification which will reproduce the real values of the effects induced in the bridge by real traffic. The static load model for bridges according to EN 1991-2 is calibrated for bridges with width less than 42m and length less than 200m. Calibration of traffic models for road bridges was based on real traffic data recorded in two experimental campaign performed in Europe between 1980 and 1994,and mainly on the traffic recorded in May 1986 in Auxerre on the motorway Paris – Lyon (Pietro Croce et al, 2010).

In EN 1991-2, four load models are considered for vertical loads and they are;

- Load Model 1 (LM1): This generally reproduces traffic loads which are to be taken into account for global and local verifications. It is made up of concentrated loads and uniformly distributed load.
- Load Model 2 (LM2): This load model reproduces effects on short structural members. It is comprised of a single axle load on a specific rectangular tire contact areas.
- Load Model 3 (LM3): Special vehicles to be considered on request, in transient design situations. It represents abnormal vehicles not complying with national regulations on weight and dimensions of vehicles.
- Load Model 4(LM4): Crowd loading

Load Model 1

The Load Model 1 which represents the effects of normal traffic comprises of tandem axles (TS) superimposed over a uniformly distributed load (UDL) which its intensity remains constant with the loaded length. The model is very different from Type HA loading given in BD37. Type HA loading consists of a uniformly distributed load, the intensity which varies with the loaded length, and a constant Knife Edge Load (KEL) of 120 k N. There are also lane factors for different lengths which account for simultaneity of loading in adjacent lanes as a function of loaded length. Euro code (EN 1991-2) load model also differs with BD37 in the way that the carriageway is divided into notional lanes (Atkins Highways and Transportation, 2004). In EN 1991-2, the notional lane width is constant at 3.0m except for a small range of carriageway width between 5.4m and 6.0m, when the lane width varies from 2.7m to 3.0m. (See Table 1.0)

Table 1.0: Subdivision of carriageway into notional lanes (Table 4.1 BS EN 1991-2:2003)

Carriageway width (w)	Number of notional lanes (n)	Width of notional lane	Width of the remaining area
$w < 5.4$m	1	3m	$w - 3$m

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5.4 ≤ *w* < 6m	2	0.5*w*	0
6m ≤ *w*	Int(*w*/3)	3m	*w* – (3 × n)

The characteristics of the Load Model 1 according to EN 1991-2 are as shown below in Table 2.0;

Table 2.0: Load Model 1 Characteristic Values (Table 4.2 BS EN 1991-2:2003)

Position	**Tandem Axle Load Q_{ik} (kN)**	**UDL q_{ik} (kN/m^2)**
Notional lane 1	300	9.0
Notional lane 2	200	2.5
Notional lane 3	100	2.5
Other notional lanes	0	2.5
Remaining Area	0	2.5

The distribution and application of Load Model 1 on a typical 3 lane carriageway is as shown in Figure 1 below.

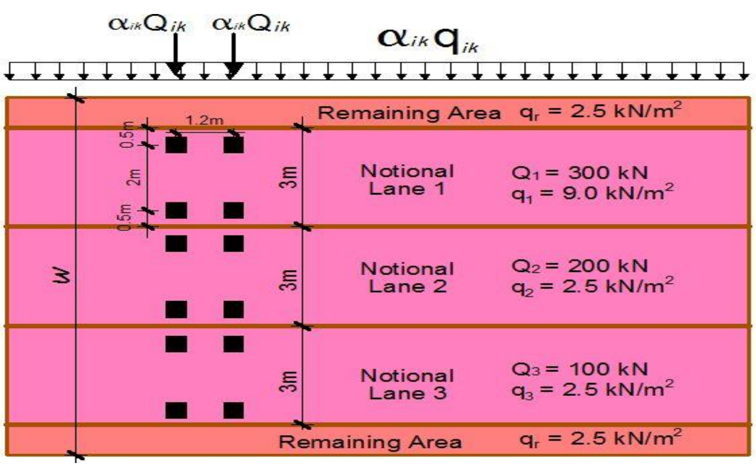

Figure 1: Application of Load Model 1 on a deck with 3 notional lanes

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Typical rules for Applying Load Model 1

1. In each notional lane, only one tandem system should be considered, situated in the most unfavourable position.
2. The tandem system should be considered travelling in the longitudinal axis of the bridge.
3. When present, the tandem system should be considered in full i.e with all its four wheels
4. The UDL's are applied longitudinally and transversally on the unfavourable parts of the influence surface.
5. The two load systems can insist on the same area.
6. The dynamic impact factor is included in the two load systems
7. When static verification is governed by combination of local and global effects, the same load arrangement should be considered for calculation of local and global effects.

According to Ellinget al (1985), refined methods of analysis for analysing highway bridge superstructures and determining girder moments and loads can be classified into six categories:

1. Orthotropic plate theory methods
2. Harmonic analysis methods
3. Grillage analogy methods
4. Finite element methods
5. Finite strip methods, and
6. Folded plate methods

Traffic loads on bridge decks are distributed according to the stiffness, geometry and boundary conditions of the deck (Ryall, 2008). In 1946 and 1950, Guy on and Massonet respectively made first attempts to simplify the method for analysing bridge decks using the orthotropic plate theory. They pioneered the principle of distribution coefficients which involved the distribution of live loads to a particular beam as a fraction of the total imposed load (Ryall, 2008). The work of Guy on and Massonet gave rise to the Guyon-Massonet method of analysis. The basic assumption of the distribution coefficient is that the distribution pattern of longitudinal moments, shears, and deflections across a transverse section is independent of the longitudinal position of the load and the transverse section considered (Bakht and Jaeger, 1985, Ryall, 1992).

The harmonic analysis method was developed in the 1950's by Hendry and Jaeger. In this method, loads are distributed to the individual girders as though the slab were a continuous beam over non-deflecting supports, and it considers the same flexural and torsional rigidities as the orthotropic plate analysis, but neglects the torsional rigidity in the transverse direction (Elling et al, 1985). The loading is expressed as a harmonic series or Fourier sine series. Expressions for shear, moment, slope, and deflection are found by successive integration of this load series. Girder bending moments are determined by considering the above series in conjunction with transverse force equilibrium and slope-deflection expressions in the transverse direction (Goldberg and Leve, 1957).

Grillage analogy presents a sufficiently accurate method of analysing bridge decks for estimation of design bending moment, torsion, shears force etc, and has been adapted for use in most computer software around the world. Basically, grillage analogy method uses stiffness approach for analyzing the bridge decks (Jaggerwal and Bajpai, 2014). The whole bridge deck is divided into a number of longitudinal and transverse beams (planar grids).In the analysis, the elements of a grid are assumed to be rigidly connected, so that the original angles between elements connected together at a node remain unchanged. Both torsional and bending moment continuity then exist at the node point of a grid (Qaqish et al, 2008).
The method has proved to be reliable and versatile for a wide variety of bridge decks.

The finite element method seeks to replace a continuous type of structural problem, which is alternatively represented by a set of partial differential equations, by a set of discrete, simultaneous linear equations which may be readily solved by computer (Johnson, 2000). The discretization is achieved by sub-dividing the surface to be considered into a number of regions and so creating a set of elements and nodes (meshes). The accuracy of the results of a finite element model increases as the element size decreases (Shreedhar and Mamadapur, 2012). The required size of elements is smaller at areas where high loads exist such as location of applied concentrated loads and reactions. For a deck slab, the dividing the width between the girders to five or more girders typically yields accurate results. The aspect ratio of the element (length-to-width ratio for plate and shell elements and longest-to-shortest side length ratio for solid elements) and the corner angles should be kept within the values recommended by the developer of the computer program. Typically aspect ratio less than 2 and corner angles between 60 and 120 degrees are considered acceptable (Shreedhar and Mamadapur, 2012).

According to (Elling et al, 1985), folded plate theory can be divided into two categories: (a) the ordinary method, in which the longitudinal behaviour of the plate is calculated according to beam theory, and the transverse behaviour according to one-way theory; and (b) the stiffness method which combines slab theory and plane stress theory. The bridge is considered as an assembly of individual, elastic, isotropic rectangular plate elements interconnected at the longitudinal joints, and simply supported at the ends.

Courbon's theory is one of the popular classic methods of analysing slab and beam girders but the results obtained from it are usually very unconservative. However, it is the easiest method to apply and does not waste time at all. It was originally developed for bridge girders with series of cross beams (diaphragm) in which the cross beams are stiff enough to provide adequate lateral stiffness. By implication, the application of the method requires that the cross beams will have a depth not less than 75% of the main longitudinal girders (Raju, 1986). It also requires that the span to width ratio of the bridge will be greater than 2 but less than 4. According to Wuzaka (2014), the mechanical model allowing the analysis of the behaviour of the cross-section of the span subjected to a force (P) is assumed in the form of an infinitely stiff beam with elastic Winkler type supports. The cross-sectional deformation after loading (Figure 2c) is equal to the

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symmetric part of the deformation after loading (Figure 2a) plus the asymmetric part of the deformation after loading (Figure 2b).

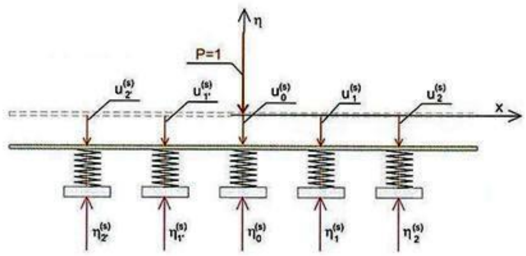

Fig. 2(a)

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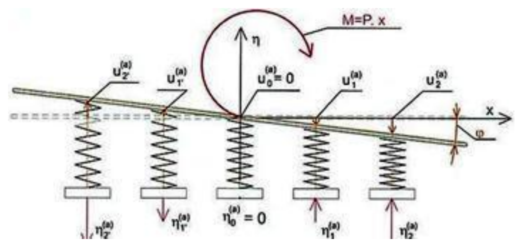

Fig. 2(b)

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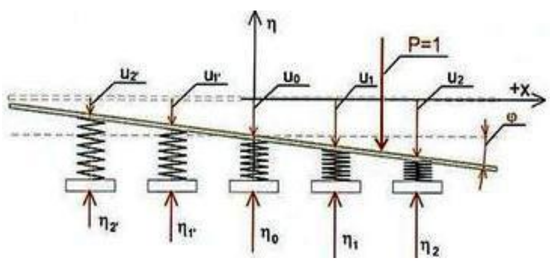

Fig. 2(c)

Figure 2: Mechanical Model of Courbon's theory (Wuzaka, 2014)

When the beams are equally spaced and geometrically equal, the reaction factor from Courbon's theory for external beams is given by (Pietro Croce et al, 2010);

Ri = $P\left[\frac{1}{n} + \frac{6e_1}{n(n+1)S}\right]$ --------------------------- (1)

Where; Ri = Reaction factor
P = Applied load
n= Number of longitudinal beams
S = Spacing of longitudinal beams
e_1 = Eccentricity of load with respect to the centroidal axis of the bridge deck

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If we set P = 1.0, we can obtain the influence line pertaining to each beam. While this theory is not applicable to the form of the bridge deck we are considering, but due to its ease of usage, we are going to apply it, and then compare the answers we get with answers from FEM and grillage analysis. Attempt will be made to calibrate the answers gotten so that we can use it as a quick check when carrying out analysis. In this paper, the maximum internal stresses (longitudinal moment and shears) developed on the exterior girders under Load Model 1 are investigated using finite element model, grillage analogy, and Courbon's theory. Then we will calibrate Courbon's theory with a correction factor so that we can always use it as a quick manual check during analysis of bridge decks.

Modelling and Methodology

In the example considered in this paper, let us look at the bridge models shown in Figures3 to 5. The total width of the bridge deck is 10.1m, while the width of the carriageway (w) is 7.2m. We consider the bridge under different girder support conditions; three, four, and five, and also under varying span lengths of 15m, 20m, and 25m. The geometry of the girders and the deck slab remained constant throughout the whole analysis. There are no cross-girders or diaphragms in the bridge models considered.

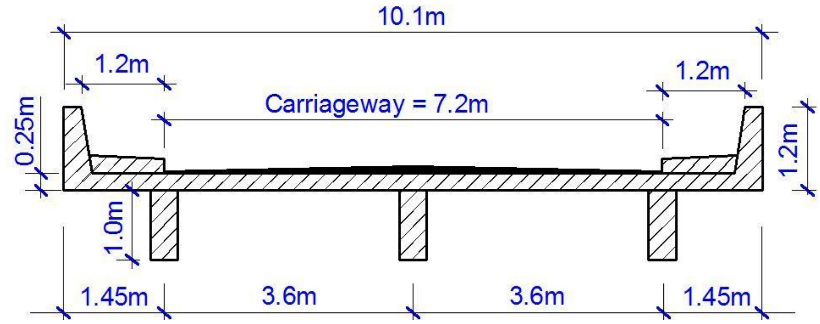

Figure *3*: Model 1 of the bridge system (3 girders)

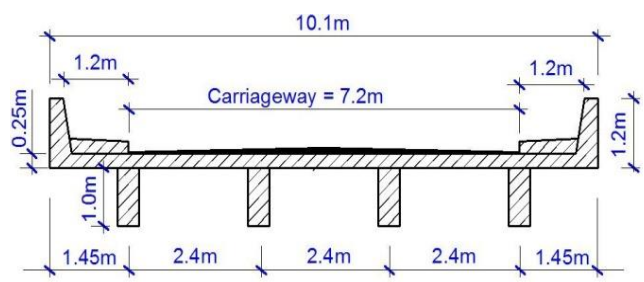

Figure 4: Model 2 of the bridge system (4 girders)

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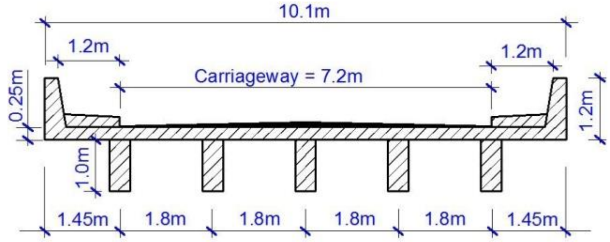

Figure 5: Model 3 of the bridge system (5 girders)

Loading

In this paper, the effect of Load Model 1 on the most exterior girder was investigated by arranging the load models in such a way as to maximise its effect on the exterior girder (see Figure 6 to see the arrangement model adopted and the position of the notional lanes). In this bridge model, the position of the exterior girder coincides with the position of the raised kerbs. Also according to EN 1991-2, traffic loads should be applied on the carriageway longitudinally and transversally in the most adverse position, according to the shape of the influence surface in order to maximise the considered load effect. In this design example, we are trying to maximise the load on the exterior longitudinal girder (see figure 7).

Width of carriageway (w) = 7.2m

Since w > 6.0m; Number of notional lanes (n) = $\text{int}\left[\frac{w}{3}\right] = \text{int}\left[\frac{7.2}{3}\right] = 2.0$

Width of remaining area = $w - (3 \times n) = 7.2 - (3 \times 2) = 1.2$m

Also, a uniformly distributed pedestrian variable load of 5 k N/m^2(clause 5.3.2.1 BS EN 1991-2:2003) was applied on one of the sidewalks to maximise the effects on the most exterior girder (Girder 1 taken as a case study). See the full loading on Figure 8. This same loading arrangement was adopted for all the different models and a little consideration will show that the position of girder 1 will never change in all the analysis. For global verifications, only Load Model 1 has been considered. All adjustment factors were taken as $\alpha Q_i = \alpha q_i = 1.0$.

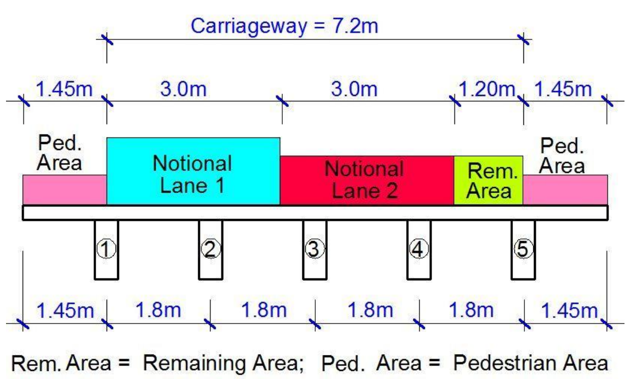

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Figure 6: *Division of the carriageway into notional lanes and typical load* values

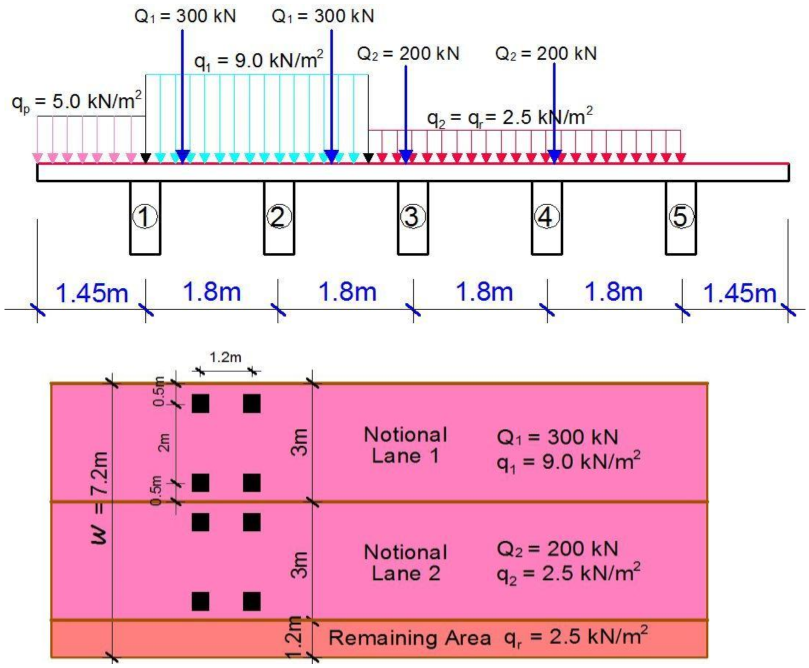


Figure 7: Application of Load Model 1 to maximise the effects on longitudinal beam 1

Analysis methods

Finite Element Analysis

The finite element model of the bridge was carried out on Staad Pro v8i using fine meshes for the plates. Each plate has a rectangular dimension of 0.1m × 0.24m. This is an incentive for more accurate results. The longitudinal beams were modelled as T-beams and this can be seen in Figure 8.

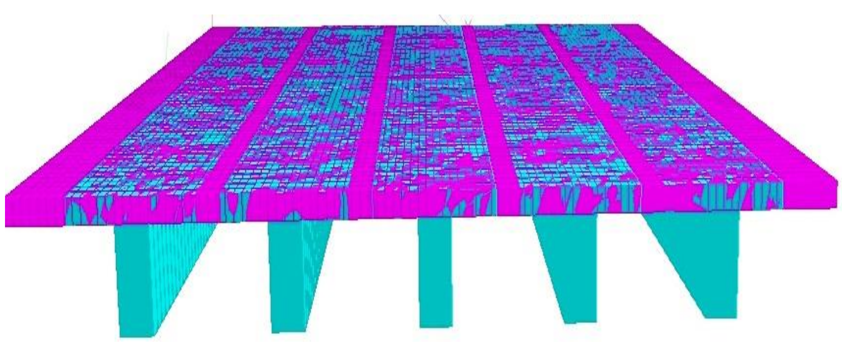


Figure 8: Rendered 3DFinite Element Model of Model 3 on Staad Pro V8i

Grillage Analysis

Different grillage models were used to represent the various bridge deck configurations and the analysis was carried out using *Staad Pro v8i* software. A rendered 3D grillage model of the bridge deck (model 3) is as shown in Figure 11, while the analysis model is shown in Figure 10. The uniformly distributed loads were applied on the longitudinal beams; while the concentrated loads (tandem loads) were applied on the nodes based on their location. The nodal loads on the grillage can be seen in Figure 10, and the way it was generated is shown in the Figure 9 below. See Table 3.0 for the computation process.

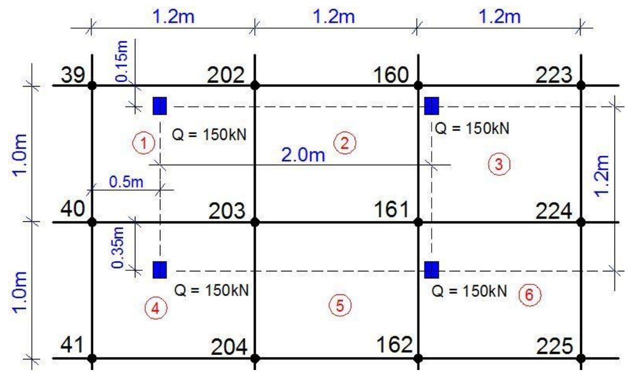


Figure 9: Application of Concentrated loads to the grillage model

The concentrated load Q = 150 kN at square 1 (see Figure 1.9) is distributed to the nodes 39, 202, 40 and 203, according to their coordinates as shown in the table below;

Table 3.0: Distribution of tandem loads to the various nodes

Plate 1 (Q = 150 kN)	**Plate 3 (Q = 150 kN)**	**Plate 4 (Q = 150 kN)**	**Plate6 (Q = 150 kN)**
P_{39} = 74.375 kN	P_{160} = 116.875 kN	P_{40} = 56.875 kN	P_{161} = 116.875 kN
P_{202} = 53.125 kN	P_{161} = 20.625 kN	P_{41} = 30.625 kN	P_{162} = 48.125 kN
P_{40} = 13.125 kN	P_{223} = 10.625 kN	P_{203} = 40.625 kN	P_{224} = 8.125 kN
P_{20}3 = 9.375 kN	P_{224} = 1.875 kN	P_{204} = 21.875 kN	P_{225} = 4.375 kN

For example, the total load transferred to node 40 (∑P40) is given by 13.125 + 56.875 = 70 KN. This method of distribution has been adopted in the entire grillage analysis.

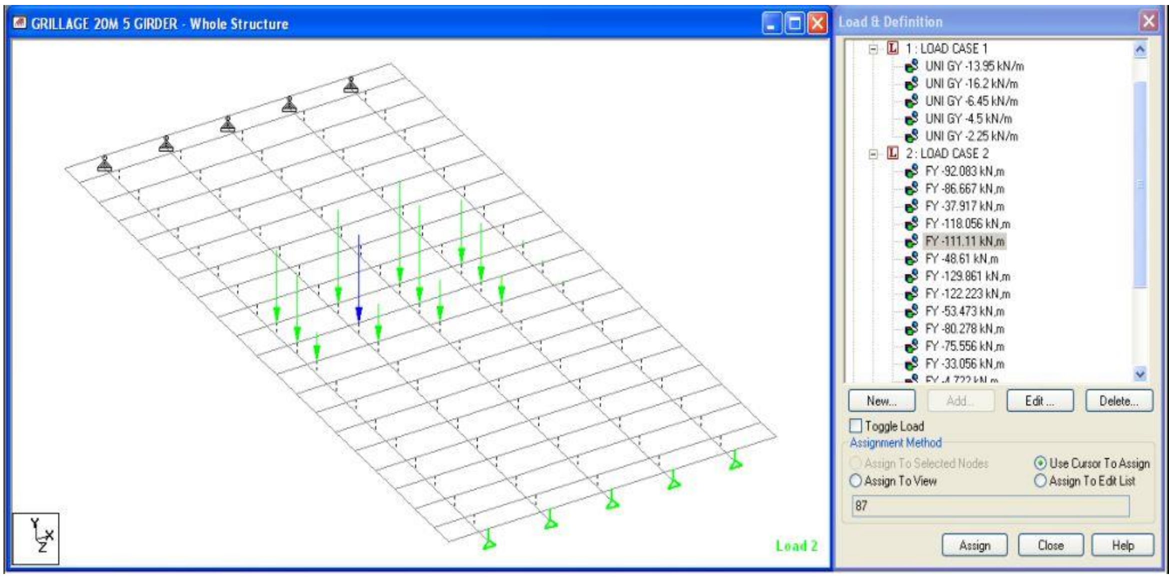
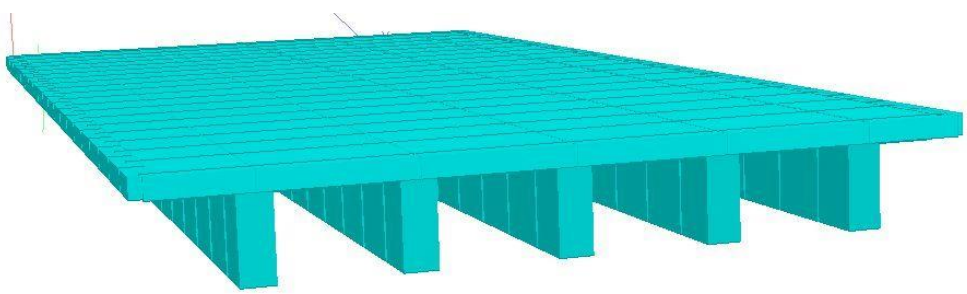


Figure 10: Grillage model (for model 3) with nodal concentrated loads on Staad Pro Software



The 3D rendering of the grillage model on Staad Pro is as shown in Figure 2.1.

Figure 11: 3D rendering of grillage model 3 on Staad Pro Software

Courbon's Method

We can verify that the reaction factor for girder number 1 when P = 1.0 is located at girder one, can be easily calculated using equation 1 for any number of girders. For the model with five girders, the calculation process is shown below. The influence diagram is shown in Figure 12.

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$R_1 = 1.0\left[\frac{1}{5} + \frac{6 \times 3.6}{5(5+1) \times 1.8}\right] = 0.60$

$R_2 = 1.0\left[\frac{1}{5} + \frac{6 \times 1.8}{5(5+1) \times 1.8}\right] = 0.40$

$R_3 = 1.0\left[\frac{1}{5} + \frac{6 \times 0}{5(5+1) \times 1.8}\right] = 0.20$

$R_4 = 1.0\left[\frac{1}{5} - \frac{6 \times 1.8}{5(5+1) \times 1.8}\right] = 0$

$R_5 = 1.0\left[\frac{1}{5} + \frac{6 \times 3.6}{5(5+1) \times 1.8}\right] = -0.2$

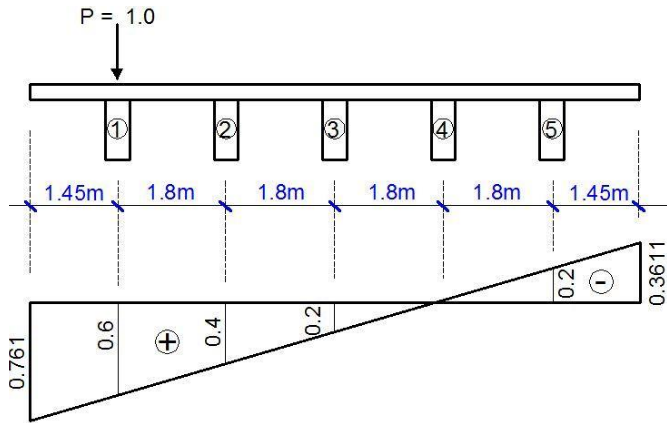

Figure 12: Influence diagram of 5 girder deck from Courbon's

We can then place the externally applied load on the influence load distribution diagram. This is shown in Figure 13.In this case; we only consider the onerous part of the influence diagram, which implies that we will only consider the positive part of the diagram.

Reading from the influence diagram for the concentrated loads (see Figure 13);

Q_T = (600 × 0.4335) + (200 × 0.211) = 302.3 kN

For uniformly distributed loads;

q_T = (5 × 1.45 × 0.6805) + (9 × 3 × 0.4335) + (2.5 × 0.3183) = 17.43 kN/m

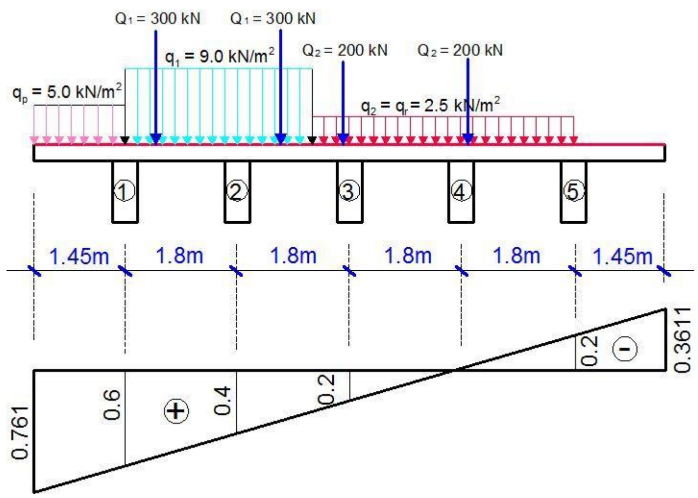

Figure 13: Application of external load on the influence diagram

We can now apply the concentrated load and uniformly distributed loadongirdernumber 1, modelled as a simply supported beam structure. See Figure 14.

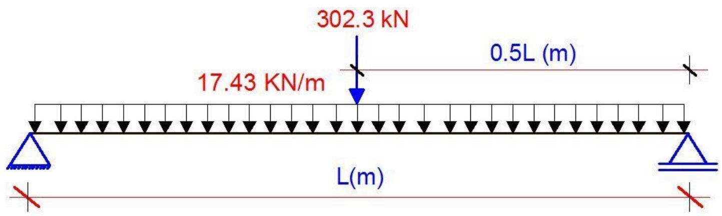

Figure 14: Application of load values derived from influence diagram on girder No 1 (Model 3)

Results of Analysis and Discussion

From the finite element analysis, grillage analogy, and Courbon's method, the maximum longitudinal moment and corresponding shear force due to the LM1 on the exterior girder are shown in Table 4.0 to 6.0 below.

Table 4.0: Results from finite element analysis

Length of Span	3 GIRDERS		4 GIRDERS		5 GIRDERS	
	Moment (kN.m)	Shear (kN)	Moment (kN.m)	Shear (kN)	Moment (kN.m)	Shear (kN)
15m	1419.245	338.585	1132.250	270.430	970.695	231.538
20m	2105.823	410.266	1665.502	330.151	1433.986	295.106
25m	2796.709	470.539	2248.720	390.565	1872.376	337.925
30m	3485.761	537.248	2788.677	451.233	2329.213	399.312

Table 5.0: Results from grillage analysis

Length of Span	3 GIRDERS		4 GIRDERS		5 GIRDERS	
	Moment (kN.m)	Shear (kN)	Moment (kN.m)	Shear (kN)	Moment (kN.m)	Shear (kN)
15m	1554.734	346.805	1303.074	278.088	1141.496	233.687
20m	2322.140	421.944	1887.652	327.148	1691.655	285.631
25m	3106.193	475.834	2531.246	382.437	2298.172	349.018

30m	3879.663	544.690	3133.678	431.567	2867.200	402.681

Table 6.0: Results from Courbon's method

Length of Span	3 GIRDERS		4 GIRDERS		5 GIRDERS	
	Moment (kN.m)	Shear (kN)	Moment (kN.m)	Shear (kN)	Moment (kN.m)	Shear (kN)
15m	2425.57	417.87	2039.69	351.17	1623.95	281.90
20m	3548.97	480.84	2983.63	403.98	2383.19	325.49
25m	4829.79	543.82	4059.58	456.00	3251.40	369.07
30m	6268.01	606.78	5267.55	509.59	4228.13	412.60

A little observation of the results will reveal that finite element analysis gave the most economical results in terms of bending moment and shearing forces. The results also indicate that the difference in the results between the longitudinal bending moment from finite element analysis method and grillage analysis increases with the number of girders and length of span. The greatest difference in the results of the longitudinal moment was found in Model 3 (5 girders) at 25m span length with a difference of 18.527% (grillage analogy has been compared against FEM). The smallest value of difference was found in Model 1 (3 girders) at 15m span length with a difference of 8.714%.

In terms of shear force, the difference in results between grillage and finite element analysis were very small when both values were compared. The highest difference in the values when shear grillage results was compared with FEM shear results was -3.3172% which was found at Model 3 (20m span length). The approximate nature of both FEM and grillage analysis meant that variation in results obtained can occur from reasons stemming from various causes (see Table 7.0 for comparison results). However, a plot of variation of internal stresses with span length for all the methods employed showed a very linear relationship for both bending moment and shearing force (see Figures 15 and 16).

The difference obtained when using Courbon's method was very wide (about 40% for longitudinal moment when compared against FEM), and this can also be justified, given that the method was applied here just to see if we can calibrate the results from it for checking results from finite element analysis, since it is a very cheap and fast method.

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Table 7.0: Comparison of results between FEM and grillage analysis

Length	FEM and GRILLAGE (3 girders)		FEM and GRILLAGE (4 girders)		FEM and GRILLAGE (5 girders)	
	Moment	**Shear**	**Moment**	**Shear**	**Moment**	**Shear**
15m	8.714%	2.370%	13.104%	2.753%	14.962%	0.9196%
20m	9.315%	2.767%	11.768%	-0.918%	15.323%	-3.3172%
25m	9.963%	1.1127%	11.161%	-2.125%	18.527%	3.1783%
30m	10.153%	1.3663%	11.001%	-4.556%	18.763%	2.404%

The variation of longitudinal bending moment for Model 1 (3 Girders) using the three different methods for the varying spans is shown in Figure 15.

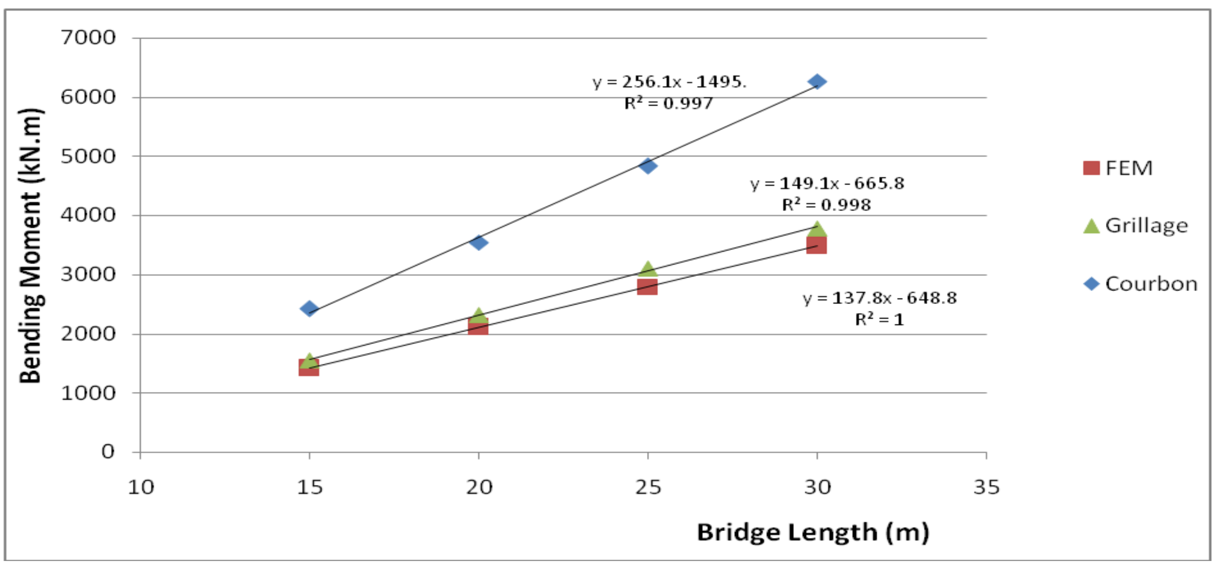

Figure 15: Variation of longitudinal bending moment for 3 Girder Bridge using different methods

Similarly for the variation of shear force for 3 girders model is shown in Figure 16;

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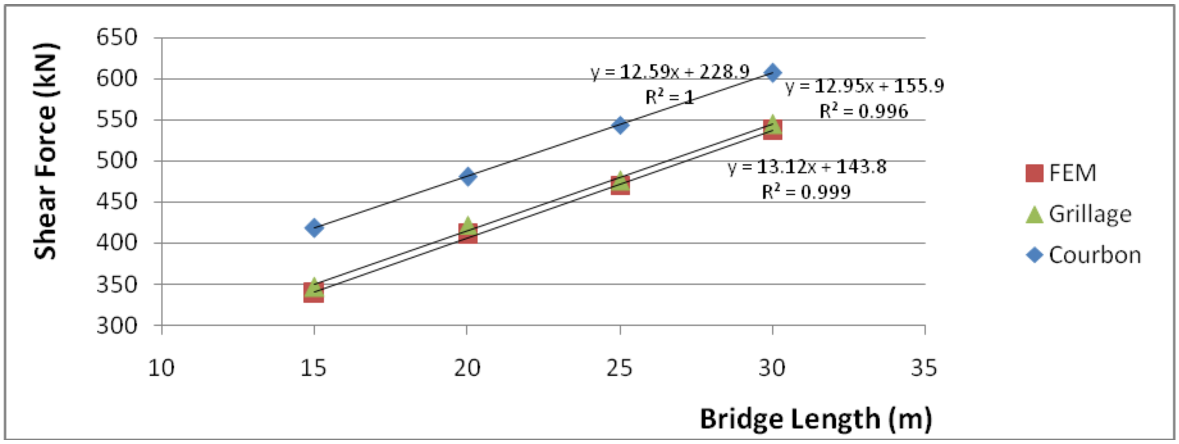

Figure 16: Variation of shear force for 3 Girder Bridge using different methods

For the whole models represented, the full regression equations for moment and shear are as shown in Table 8.0.

Table 8.0: Regression equations for bending moment and shear force across the different models

Number of girders	Method	Moment Regression Equation	Shear Regression Equation
3	FEM	$M = 137.8x - 648.8$ ($R^2 = 1.0$)	$Q = 13.12x+143.8$ ($R^2 = 0.999$)
	Grillage	$M = 149.1x - 665.8$ ($R^2 = 0.998$)	$Q= 12.95x+155.9$ ($R^2 = 0.996$)
	Courbon	$M = 256.1x - 1495.0$ ($R^2 = 0.998$)	$Q = 12.59x + 228.9$ ($R^2 = 1.0$)
4	FEM	$M = 111.0x - 539.8$ ($R^2 = 0.999$)	$Q = 12.05x+89.32$ ($R^2 = 1.0$)
	Grillage	$M = 122.7x - 547.0$($R^2 = 0.999$)	$Q= 10.31x+122.7$ ($R^2 = 0.999$)
	Courbon	$M = 215.1x - 1254.5$($R^2 = 0.997$)	$Q= 10.54x + 192.9$ ($R^2 = 1.0$)

5	FEM	M = 90.27x – 379.7 (R^2 = 0.999)	Q = 10.92x+70.20 (R^2 = 0.994)
	Grillage	M = 115.6x – 603 (R^2 = 0.999)	Q= 11.40x+61.08 (R^2 = 0.998)
	Courbon	M = 173.6x – 1034.6R^2 = 0.996)	Q = 8.713x + 151.2 (R^2 = 1.0)

If we attempt to compare the results from Courbon's theory with finite element analysis, we can obtain what can be described as a calibration factor (C_f), which can be used to factor Courbon's method results to fit within the range of finite element analysis results. The variation of the longitudinal bending moment results between Courbon's method and Finite Element Analysis (which is the calibration factor) can be shown by the curves in Figure 17 below.

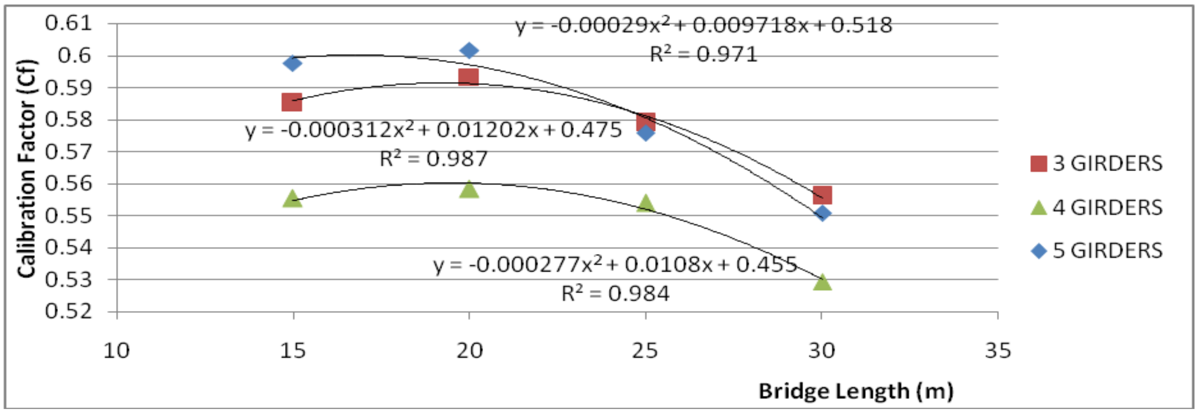

Figure 17: Variation of bending moment calibration factor between FEM and Courbon's

The calibration equations for bending moment are explicitly given in Table 9.0 below;

Table 9.0: Calibration factor for Courbon's method longitudinal bending moment

Number of girders	**Regression Calibration Equation for moment**	R^2
3	$Cf_m = -0.000312x^2 + 0.01202x + 0.4757$	0.9872

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4	$Cf_m = -0.000277x^2 + 0.01084x + 0.454$	0.9847
5	$Cf_m = -0.00029x^2 + 0.009718x + 0.5187$	0.9712

Therefore having obtained the calibration factors for moment as a function of the span of the bridge, it means that within the specified range in this study, any bending moment obtained using Courbon's method ($M_{Courbon}$) can be calibrated using the calibration factor for moment (Cf_m) to obtain an approximate equivalent FEM bending moment value (M_f). We can verify that the calibration factor for moment reduces as the bridge span increases.

Hence, $M_f \approx M_{Courbon} \times Cf_m$ ---------------------------- (2)

We can also attempt such calibration equation for shear and the plot of the variation is shown in Figure 18.

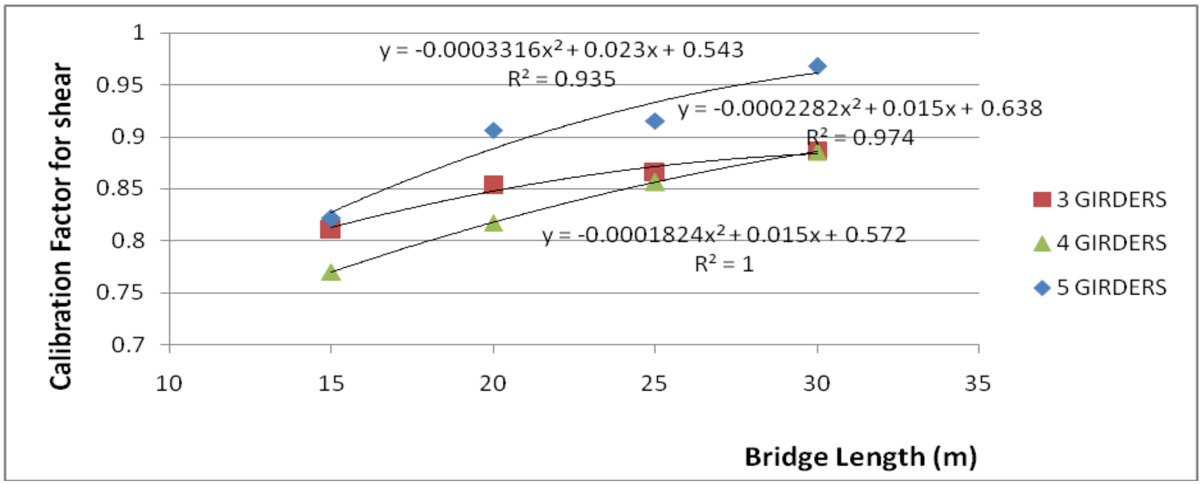

Figure 18: Variation of calibration factor for shear forces between FEM and Courbon's method

The calibration equations for shear are explicitly given in Table 10.0 below;

Table 10.0: Calibration factor for Courbon's method shear forces

Number of girders	**Regression Calibration Equation for Shear**	**R^2**
3	$Cf_q = -0.0002282x^2 + 0.01502x + 0.6383$	0.9748

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4	$Cf_q = -0.0001824x^2 + 0.01592x + 0.721$	1.000
5	$Cf_q = -0.0003316x^2 + 0.02389x + 0.5435$	0.9352

Hence, $Q_{fem} \approx Q_{Courbon} \times Cf_q$ ---------------------------- (3)

Application Example

To show how the calibration factor can be applied for bending moments, let us consider the work of (Praful and Hanumat, 2015) on a 3 girder bridge supporting Class AA tracked vehicle (IRC code). The bridge deck has five (5) cross girders.

The results from his analysis for the live load are as shown in Table 11.0;

Table 11.0: Class AA tracked vehicle live load analysis results (Praful and Hanumat, 2015)

Span	Bending Moment (kN.m)	
	Courbon	FEM
16m	2730.03	1639.381
20m	3929.14	2141.054

For span x = 16m; Calibration factor, $Cf_m = Cf_m = -0.000312(16)^2 + 0.01202(16) + 0.4757 = 0.5881$

Hence, approximate FEA moment $(M_f) \approx Cf_m \times M_{Courbon} = 0.5881 \times 2730.03 = 1605.531$ KNm (compare with 1639.381 KNm)

For span x = 20m; $Cf_m = -0.000312(20)^2 + 0.01202(20) + 0.4757 = 0.5875$

$(M_f) \approx Cf_m \times$ Mcourbon $= 0.5875 \times 3929.14 = 2308.212$ KNm (compare with 2141.054 KN. m)

Conclusion

In this paper, it has been shown as consistent with many other works, that FEM analysis yields lower and more economical results as grillage analysis for bridge decks. The longitudinal bending moment and shear forces increases as the bridge span increases, but reduces as the number of longitudinal girders increases. However, both can be conveniently employed for the purpose of analysis and design. Furthermore, if we observe the results of the calibration of internal stresses gotten from Courbon's method, we can confidently apply them on bridges with and without cross girders, and obtain very reasonable and comparable answers for internal stresses (moment and shear). Given that these factors are derived using Load Model 1 of EN 1991-2, care should be taken when applying it for other load models, or for loadings involving tandem axles only (especially for shearing forces). This is due to the difference in distribution of shearing forces for concentrated loads and uniformly distributed loads. However, for bending moment values, the difference will not be much. Hence, the calibration factors can be used for design purposes, or more especially for making very quick checks of answers gotten from FEM when analysing T-beam bridge decks within the applied range.

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