



SEISMIC PATTERN IDENTIFICATION, COINTEGRATION TEST, AND AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTIC MODELS IN THE EASTERN COAST AREAS OF TAIWAN

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ABSTRACT

Four purposes are set for this paper. The first one is for the seismic pattern identification of monthly number of earthquakes of three counties on the eastern coast of Taiwan - Yilan, Hualien and Taitung. The monthly number of earthquakes were obtained from the seismic archive of the Central Weather Bureau (CWB) of Taiwan from January 1995 to June 2017, totally 22.5 years (270 months). The versatile autoregressive integration moving average (ARIMA) model is used to identify seismic patterns in these three areas. ARIMA(3,0,0), ARIMA(2,0,1), and ARIMA(1,0,1) models can completely satisfy the statistical theories testing procedures for Yilan, Hualien and Taitung respectively.

The second purpose of this paper is to find out whether the time series of monthly number of earthquakes in the past 270 months in Yilan, Hualien and Taitung are stationary or not. The Augmented Dickey-Fuller (ADF) unit-root testing method is used to check the stationarity of the time series, and the results show that Yilan, Hualien and Taitung all reveal to be stationary.

The third purpose is to check whether the time series of the monthly number of earthquakes in Yilan and Taitung are cointegrated with Hualien or not. The outcome shows the time series of Hualien are cointegrated with Yilan and Taitung, and no spurious effect exists in regression analyses. The regression equations show that a unit increase of earthquake in Hualien may result in an increase of 0.2151 unit earthquake in Yilan, and 0.1692 unit increase of earthquake in Taitung.

The fourth purpose is to find out whether the autoregressive conditional heteroskedastic (ARCH) variances of the monthly number of earthquakes in the three counties have ARCH(1) property or not. The ARCH test shows the volatile variances of time series for both Hualien and Yilan are heteroskedastic, but Taitung is homoskedastic.

Key Words: CWB Archive, ARCH Model, Heteroskedastic, Homoskedastic

1. Introduction

Autoregressive integrated moving average (ARIMA) method is versatile and powerful because it does not have to assume any particular pattern in the historical data. Typically, it is written as $ARIMA(p,q,r)$, where the three parameters p , q , r in the parenthesis are to be determined by the shape of time series, pattern of autocorrelation function (ACF), and partial autocorrelation function (PACF). As long as these three parameters p , q , r were found, $ARIMA(p,q,r)$ models can be used to regress the time series data. The residuals of ARIMA regression shall be saved for further ACF check. If the ACF of residuals of all autocorrelation coefficients are within $(1-\alpha)\%$ confidence interval, then it is a suitable $ARIMA(p,q,r)$ model. Where α is the level of significance, and common choices of it is 0.05 or 0.01. Experience, trial and error, and luck are needed to select p , q , r parameters. Principle of parsimony refers to the preference of simple ARIMA model over complex ones (Hanke and Wichern) shall be obeyed. The detailed processes of selection of $ARIMA(p,q,r)$ model are shown in Appendix A. As long as a suitable $ARIMA(p,q,r)$ model is selected, a short range forecast can be performed. In this study, the time series are the monthly number of earthquakes strike in Yilan, Hualien, and Taitung. These three areas are on the eastern coast of Taiwan where the seismic frequencies account for up to 78% of total amount of earthquakes of the whole of Taiwan.

The earthquake data are obtained from the Central Weather Bureau (CWB) public archive (CWB, 2017) from January 1995 to June 2017. The time period covers 22.5 years or 270 months.

A stationary variable is one that is not explosive nor tending, and nor wandering aimlessly without returning to its mean (Hill, Griffiths, and Lim). If a time series is not stationary, a danger of unrelated data may have a significant regression result. Such regressions are said to be spurious (Hill, Griffiths, and Lim; Hanke and Wichern), and should be avoided. In this study, the unit-root test Augmented Dickey-Fuller (ADF) method is used to check the stationarity of the time series of the monthly number of earthquakes from January 1995 to June 2017.

Autoregressive conditional heteroskedastic (ARCH) model was proposed by Nobel Prize winner Robert Engle to deal with the volatility of inflation (Engle; Hill, Griffiths, and Lim). In this study, ARCH is used to check the existence of the heteroskedasticity of the time series of monthly number of earthquakes in Yilan, Hualien and Taitung Counties. When the variances for all of the observations are not the same, the heteroskedasticity exists. Otherwise, if all the variances observed are the same, the time series is homoskedastic.

2. Pattern Identification of Yilan, Hualien, and Taitung

The $ARIMA(p,q,r)$ models for Yilan, Hualien and Taitung will be identified through time series plot, ACF and PACF pattern check. The time series of Yilan, Hualien, and Taitung from January 1995 to June 2017 are plotted in Figure 1.

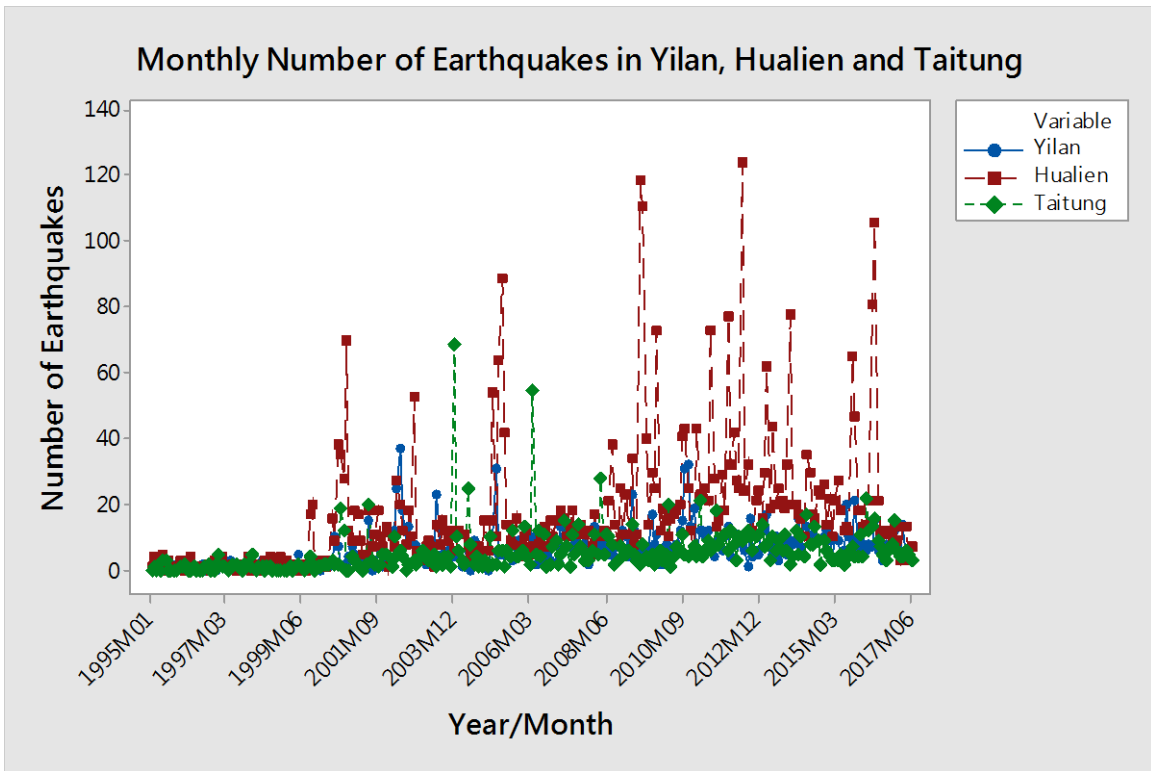


Figure 1: Monthly number of earthquakes in Yilan, Hualien, and Taitung

2.1 Pattern Identification in Yilan

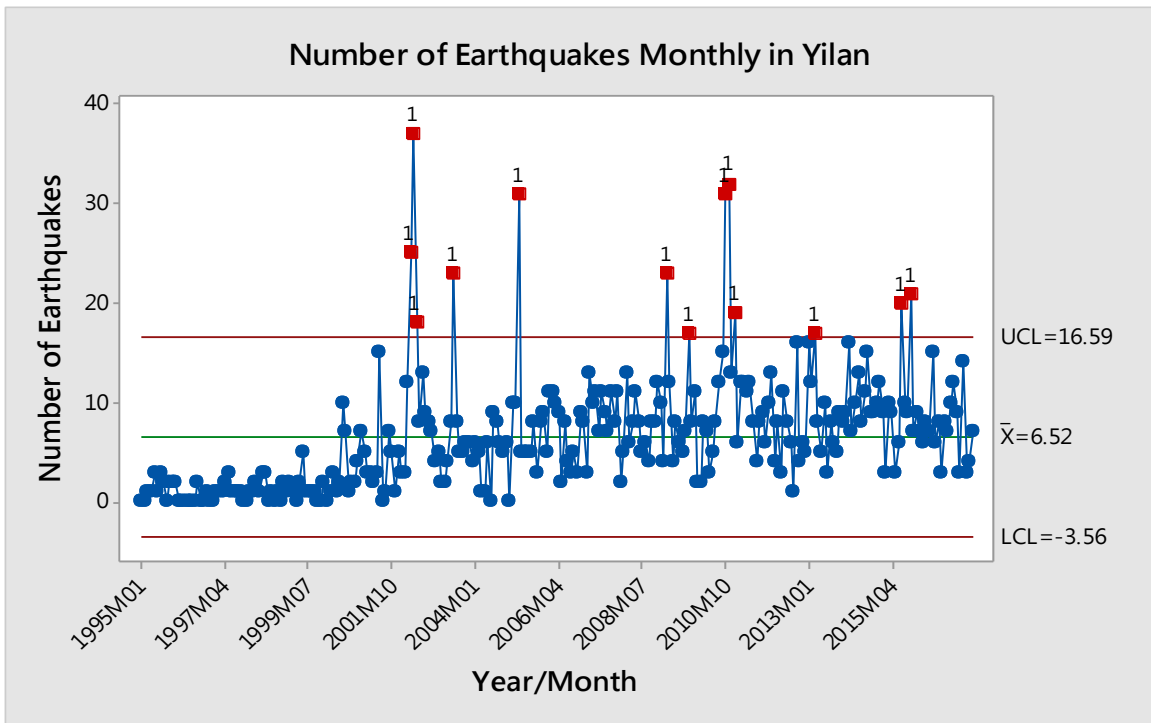


Figure 2: Monthly number of earthquakes in Yilan

From the above figure, one finds the average number of earthquakes in Yilan is 6.52. There are thirteen months with number of earthquakes above three standard errors of mean.

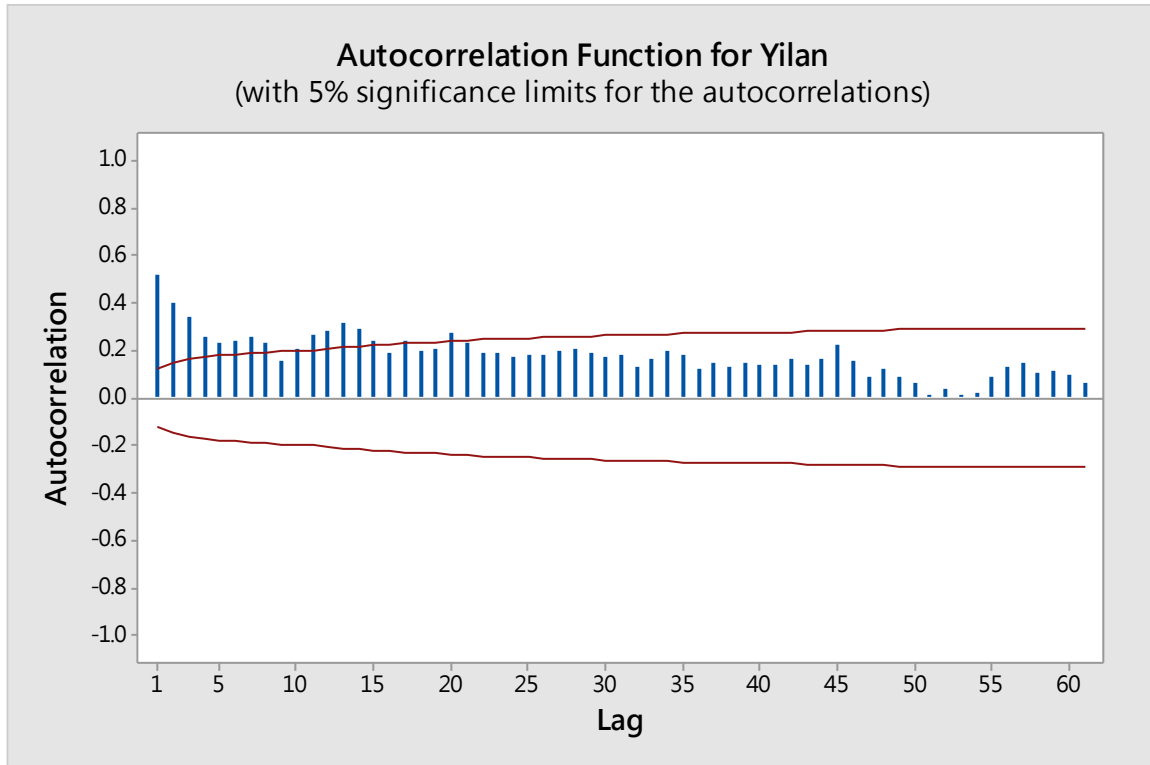


Figure 3: Autocorrelation function (ACF) of monthly number of earthquakes in Yilan

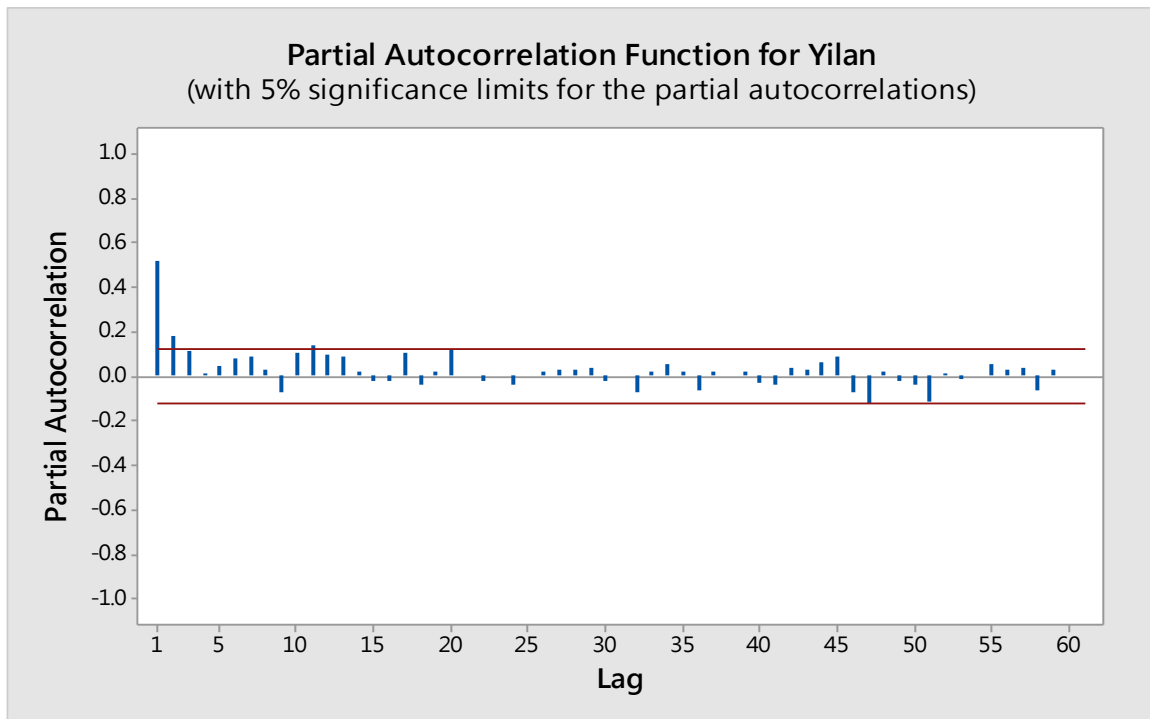


Figure 4: Partial autocorrelation function (PACF) of monthly number of earthquakes in Yilan

From autocorrelation and partial autocorrelation functions of the time series of Yilan, one finds that time series is not random because values of autocorrelation coefficients does not drop close to zero even up to 40 lags. It means the value at time Y_t correlated with value at time Y_{t-k} for any time lag k . On the other hand, if a time series is random, autocorrelation coefficients r_k for any lags are close to zero (Hanke and Wichern). Evaluated autocorrelation function and partial autocorrelation function diagrams are shown in figures 3 and 4, ARIMA(3,0,0) can be a feasible candidate for forecasting. However, the ACF of residuals have to be checked for the final confirmation.

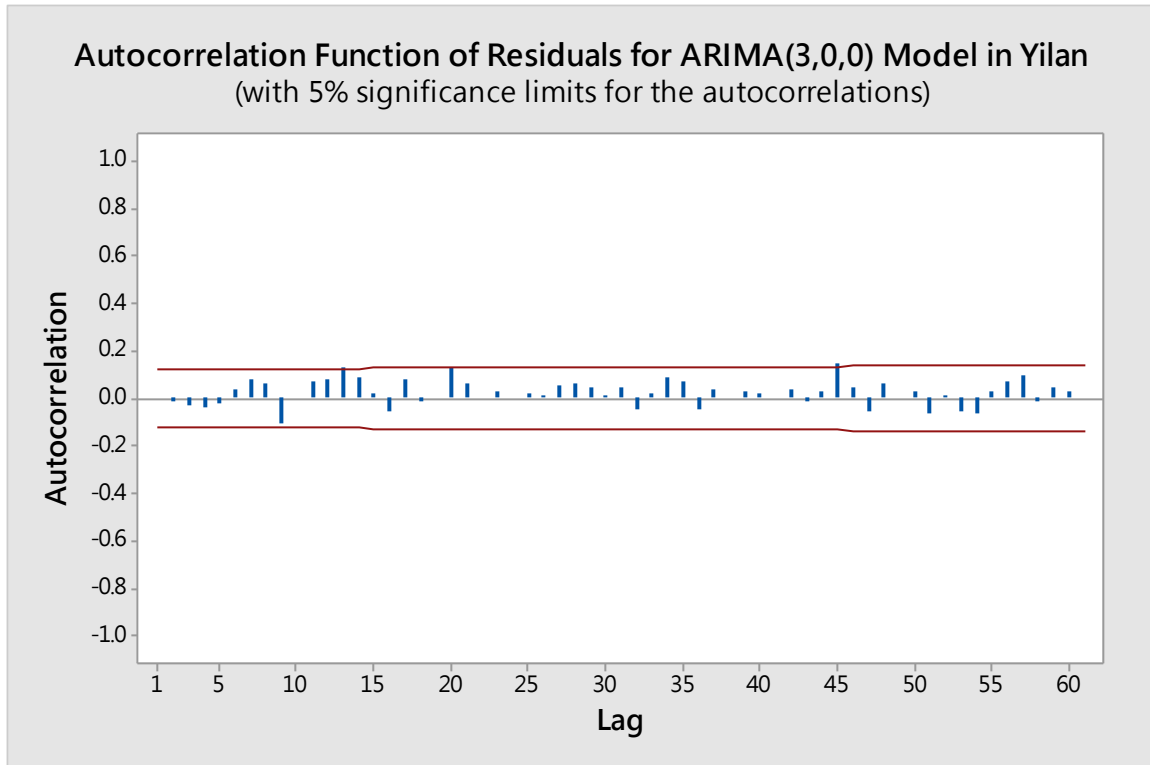


Figure 5: Autocorrelation function (ACF) of the residuals of model ARIMA(3,0,0) in Yilan

From the above figure, the ACF of the residuals of model ARIMA(3,0,0) is all in the range of two red lines, which represents 95% confidence interval. More precisely speaking, ARIMA(3,0,0) is a suitable model for monthly number of earthquakes forecasting in Yilan.

The mathematical equation of ARIMA(3,0,0) model can be expressed as:

$$Y_t = 2.2172 + 0.4074Y_{t-1} + 0.1320Y_{t-2} + 0.1165Y_{t-3} \quad (1)$$

Where, Y_t is the number of earthquakes at time t ; $Y_{t-1}, Y_{t-2}, Y_{t-3}$ are the number of earthquakes at previous time $t-1, t-2$ and $t-3$ respectively.

2.2 Pattern Identification in Hualien

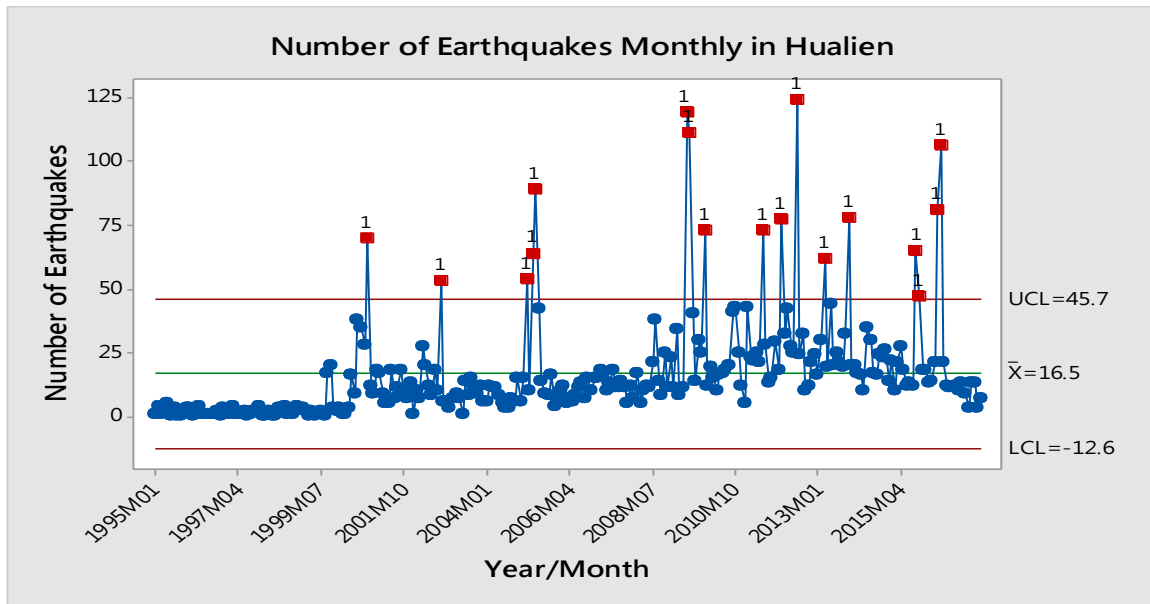


Figure 6: Monthly number of earthquakes in Hualien

From the above figure, one finds the average number of earthquakes in Hualien is 16.5. There are seventeen (17) months with the number of earthquakes above three standard errors of mean. The abrupt change in the number of earthquakes makes it difficult to forecast.

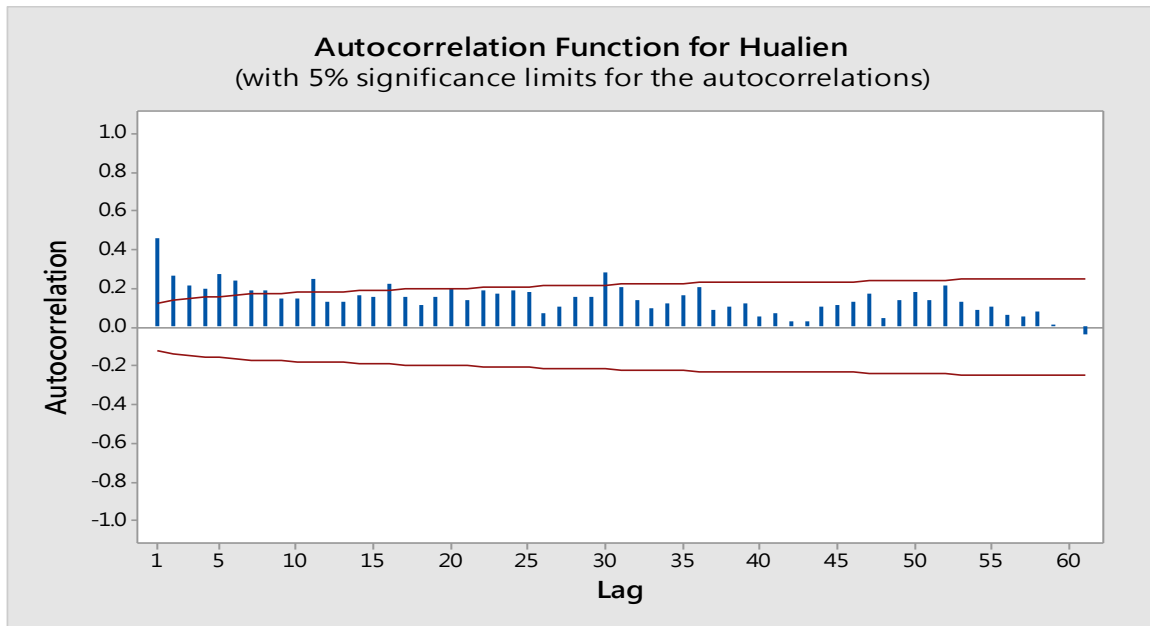


Figure 7: Autocorrelation function (ACF) of monthly number of earthquakes in Hualien

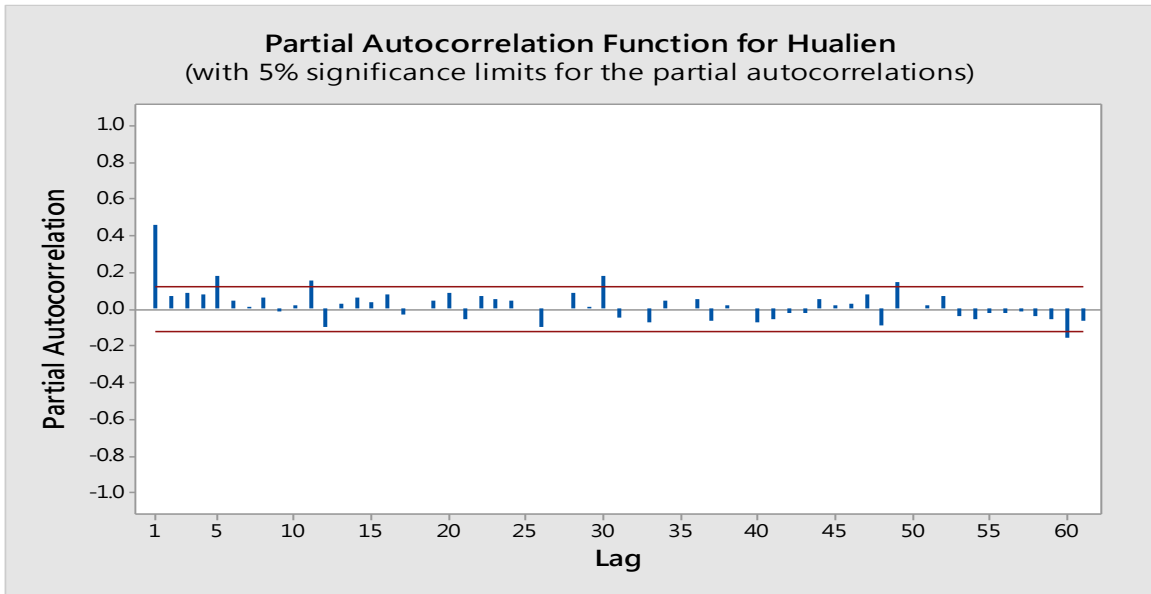


Figure 8: Partial autocorrelation function (PACF) of monthly number of earthquakes in Hualien

Following the ARIMA(p,q,r) procedures in Appendix A, the monthly number of earthquakes in Hualien can be expressed by ARIMA(2,0,1) model as:

$$Y_t = 1.3004Y_{t-1} - 0.2995Y_{t-2} - 0.9616\varepsilon_{t-1} \quad (2)$$

Where, Y_t is the number of earthquakes at time t ; Y_{t-1}, Y_{t-2} are the number of earthquakes at previous time $t-1$, and $t-2$ respectively, and ε_{t-1} is the error term at time $t-1$.

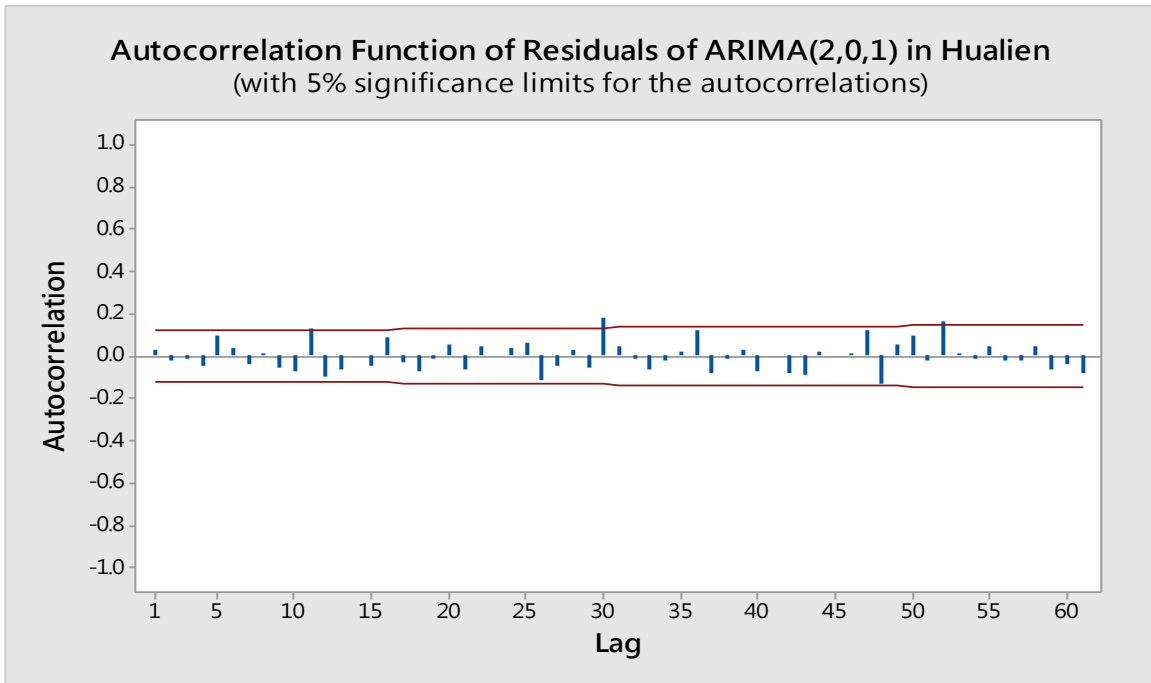


Figure 9: Autocorrelation function of residuals of ARIMA(2,0,1) model in Hualien

From the above figure, the ACF of the residuals of model ARIMA(2,0,1) is almost all in the range of two red lines, which represents 95% confidence interval. More precisely speaking, ARIMA(2,0,1) is a suitable model for forecasting the monthly number of earthquakes in Hualien.

2.3 Pattern Identification in Taitung

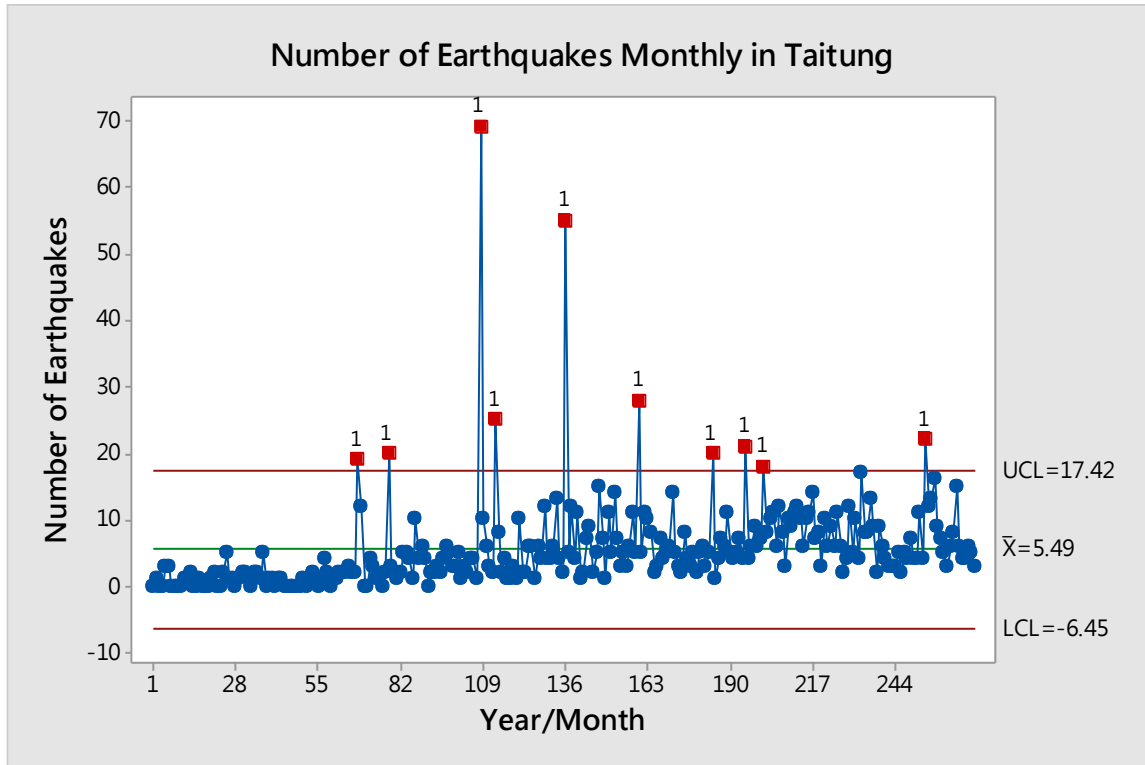


Figure 10: Monthly number of earthquakes in Taitung

From the above figure, one finds the average number of earthquakes in Taitung is 5.49. There are ten (10) months with the number of earthquakes above three standard errors of mean. The abrupt change in the number of earthquakes makes it difficult to forecast.

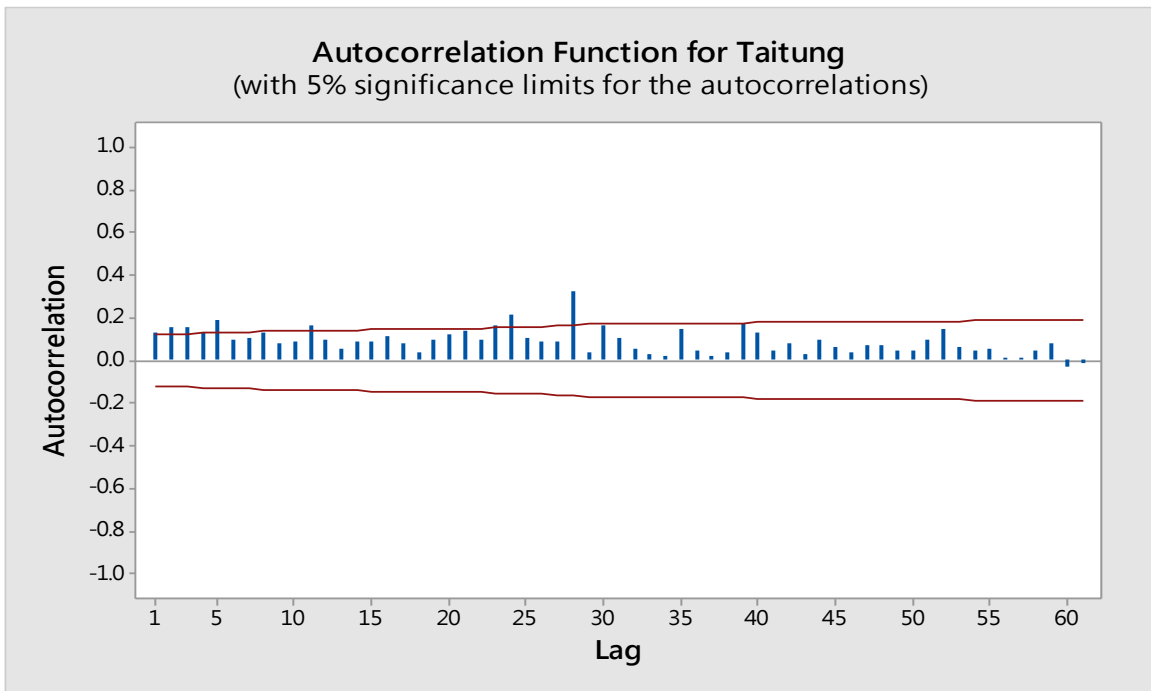


Figure 11: Autocorrelation function (ACF) of monthly number of earthquakes in Taitung

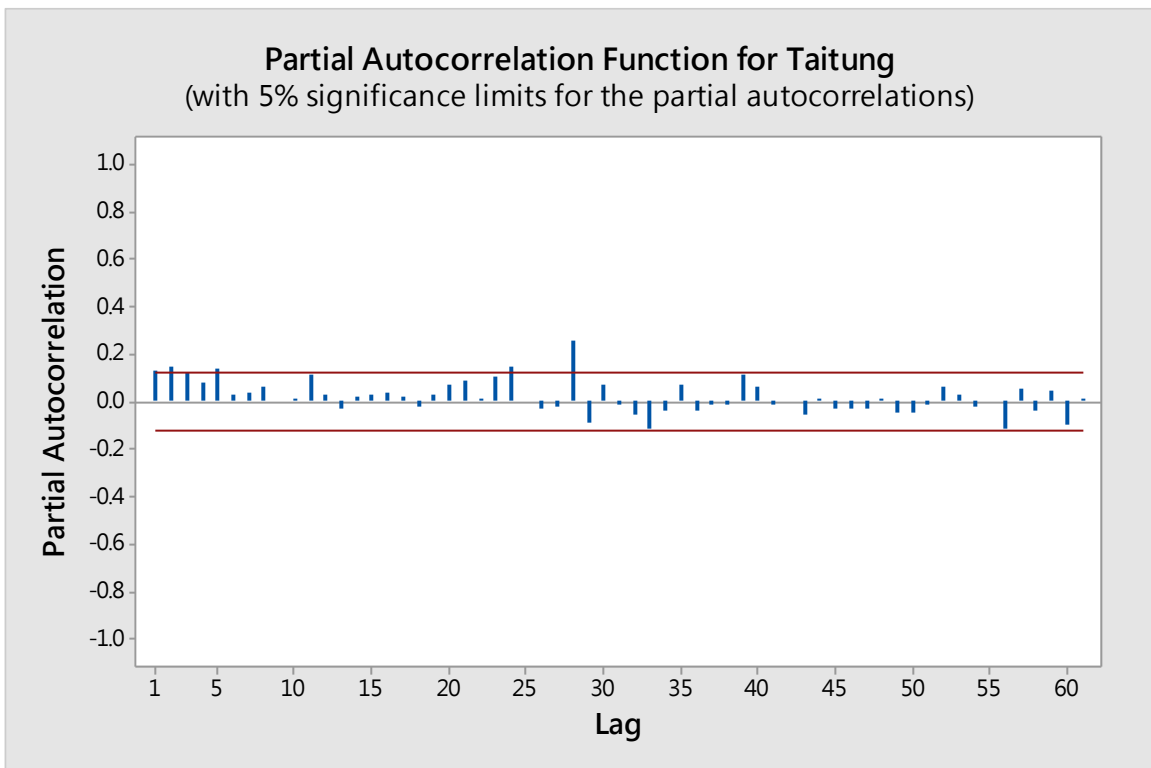


Figure 12: Partial autocorrelation function (PACF) of monthly number of earthquakes in Taitung

Following the ARIMA(p,q,r) procedures in Appendix A, the monthly number of earthquakes in Taitung can be expressed by ARIMA(1,0,1) model as:

$$Y_t = 1.0019Y_{t-1} - 0.9629\varepsilon_{t-1} \quad (3)$$

Where, Y_t is the number of earthquakes at time t , Y_{t-1} is the number of earthquakes at time $t-1$, and ε_{t-1} is the error term at time $t-1$.

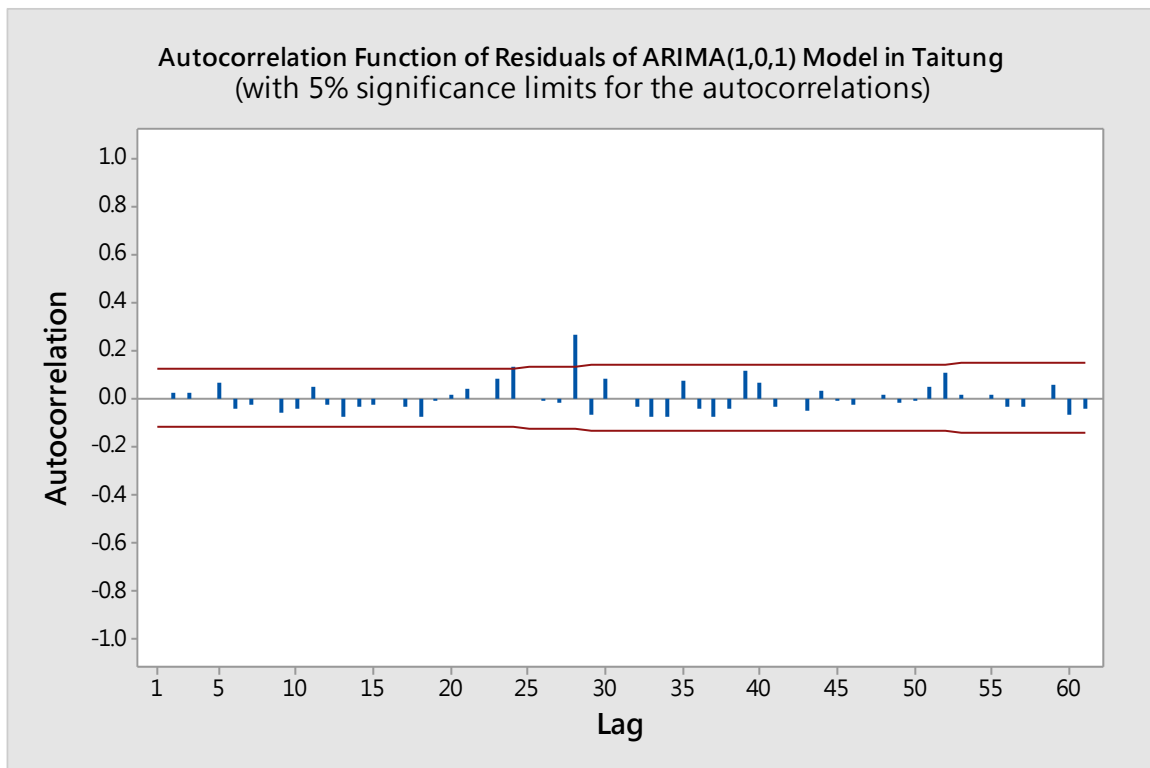


Figure 13: Autocorrelation function residuals using ARIMA(1,0,1) model in Taitung

From the above figure, the ACF of the residuals using model ARIMA(1,0,1) is almost all in the range of two red lines, which represents 95% confidence interval. More precisely speaking, ARIMA(1,0,1) is a suitable model for forecasting the monthly number of earthquakes in Taitung.

Table 1: Pattern Identification and ARIMA Equation in Yilan, Hualien, and Taitung

County	Pattern Identification ARIMA(p,q,r)	Equation
Yilan	ARIMA(3,0,0)	$Y_t = 2.2172 + 0.4074Y_{t-1} + 0.1320Y_{t-2} + 0.1165Y_{t-3}$
Hualien	ARIMA(2,0,1)	$Y_t = 1.3004Y_{t-1} - 0.2995Y_{t-2} - 0.9616\varepsilon_{t-1}$
Taitung	ARIMA(1,0,1)	$Y_t = 1.0019Y_{t-1} - 0.9629\varepsilon_{t-1}$

2.4 Checking Correctness of Proposed ARIMA models in Yilan, Hualien, and Taitung

To further check the correctness of proposed ARIMA models in the three eastern coast areas of Taiwan, the half-split scheme will be used. The half-split scheme means monthly earthquake data are roughly divided into half, and the proposed ARIMA models are used to forecast the second half.

In this study, the data from January 1995 to December 2011 were used to forecast twelve months (year 2012) and calculate errors. Next, data from January 1995 to December 2012 were used to forecast the next 12 months, and calculate errors. These procedures proceed until June 2017.

The mean absolute deviation (MAD) method (Hanke and Wichern) is used for the calculation of the average number of errors between the observed monthly number of earthquakes at time t and forecasted ones.

$$MAD = \frac{1}{n} \sum_{t=1}^n |Y_t - \hat{Y}_t| \quad (4)$$

Where Y_t is observed monthly number of earthquakes at time t , and \hat{Y}_t is the forecasted monthly number of earthquakes.

The average deviation of number in each year can be summarized as the following table:

Table 2: Mean absolute deviation (MAD) of the number of earthquakes between observed and forecasted ones in each year

Year	Yilan ARIMA(3,0,0)	Hualien ARIMA(2,0,1)	Taitung ARIMA(1,0,1)
2012	3.7	19.0	2.4
2013	6.1	15.5	3.2
2014	6.4	14.5	3.1
2015	5.0	14.2	4.5
2016	3.2	22.3	4.6
2017 (to June)	3.3	18.2	1.9
Average deviation	4.7	17.2	3.6

Although the ARIMA model in each county was built complete according to statistical theory, but the results do not seem very satisfactory. The average deviation in Hualien - 17.2 times per month is the highest among all three areas. Further study might be necessary to improve the correctness of forecasting.

3. Stationarity of Time Series in Yilan, Hualien, and Taitung

3.1 Stationarity

A stationary variable is one that is not explosive, nor trending, and nor wandering aimlessly without returning to its mean (Hill, Griffiths, and Lim). One can check the stationarity of a time series by means of visual inspection, or by more formal tests, such as unit-root tests. Dickey-Fuller, one of the unit-root tests, was used to check the stationarity of a time series in this paper. The Dickey-Fuller test has a number of variety forms, and generally referred as the Augmented Dickey-Fuller (ADF) test (Hill, Griffiths, and Lim; Hyndman and Athanasopoulos).

3.1.1 The Dickey-Fuller Critical Values

To test the hypothesis in all the cases, one can simply estimate the test equation by means of least squares and then examine the t -statistic for random walks exist or not. Unfortunately, this t -statistic no longer has the t -distribution, rather, tau (τ) statistic has to be used (Hill, Griffiths, and Lim). The critical values of tau (τ) statistic are given in Table 3.

3.1.2 Augmented Dickey-Fuller (ADF) Test

3.1.2.1 Yilan

The critical values of the Augmented Dickey-Fuller (ADF) test of the time series of the monthly number of earthquakes in Yilan are shown in Table 3.

Table 3: The critical values and Dickey-Fuller unit-root test results for Yilan

$\tau(t)$ Test statistic	1% Critical value	5% Critical value	10% Critical value
-9.217	-3.458	-2.879	-2.570
MacKinnon approximate p -value for $\tau(t) = 0.0000$			

From the above table, one finds the $\tau(t)$ test statistic $-9.217 < -2.879$ (5% critical value), the hypothesis test $H_0 : \gamma = 0$ (nonstationary) is rejected, and $H_1 : \gamma < 0$ (stationary) is not rejected. In other words, the time series of the monthly number of earthquakes in Yilan is a stationary one.

3.1.2.2 Hualien

The critical values of the Augmented Dickey-Fuller (ADF) test of the time series of the monthly number of earthquakes in Hualien are shown in Table 4.

Table 4: The critical values and Dickey-Fuller unit-root test results for Hualien

$\tau(t)$ Test statistic	1% Critical value	5% Critical value	10% Critical value
-9.919	-3.458	-2.879	-2.570
MacKinnon approximate p -value for $\tau(t) = 0.0000$			

From the above table, one finds the $\tau(t)$ test statistic $-9.919 < -2.879$ (5% critical value), the hypothesis test $H_0 : \gamma = 0$ (nonstationary) is rejected, and $H_1 : \gamma < 0$ (stationary) is not rejected. In other words, the time series of the monthly number of earthquakes in Hualien is a stationary one.

3.1.2.3 Taitung

Similarly, the ADF test for Taitung is shown in Table 5.

Table 5: The critical values and Dickey-Fuller unit-root test results for Yilan

$\tau(t)$ Test statistic	1% Critical value	5% Critical value	10% Critical value
-14.295	-3.458	-2.879	-2.570
MacKinnon approximate p -value for $\tau(t) = 0.0000$			

From the above table, one can also find the hypothesis test $H_0 : \gamma = 0$ (nonstationary) is rejected, and $H_1 : \gamma < 0$ (stationary) is not rejected. Hence, that the time series of monthly earthquakes per month in Taitung is stationary can be confirmed.

All the time series for the monthly number of earthquakes in Yilan, Hualien, and Taitung are stationary, then the worrisome spuriousness can be reduced but not completely eliminated. As long as the residuals of the regression are stationary, then the cointegration can be confirmed, and the spuriousness can be eliminated.

4. Cointegration Test

4.1 Cointegration Test of Yilan and Hualien

Generally speaking, nonstationary time-series variables should not be used in regression models to avoid the problem of spurious regression (Hill, Griffiths, and Lim). In the previous subsection, the monthly number of all the earthquakes in Yilan, Hualien, and Taitung were proved to be stationary. The spurious situation is not supposed to exist. However, for eliminating all anxieties of spurious regression, the cointegration effect is still checked. If the residuals of regression of two time-series are stationary, these two time-series are said to be cointegrated. The Augmented Dickey-Fuller (ADF) test still can be used to check the residuals of the regression, only the different critical values will be used (Hill, Griffiths, and Lim). The cointegration test of Yilan and Hualien is shown in Table 6.

Table 6: Critical values of the cointegration test

$\tau(t)$ Test statistic	1% Critical value	5% Critical value	10% Critical value
-9.78	-3.39	-2.76	-2.45
MacKinnon approximate p -value for $\tau(t) = 0.0000$			

From the above table, one finds the $\tau(t)$ test statistic $-9.78 < -2.76$ (5% critical value), the hypothesis test H_0 : the series are not cointegrated (residuals are nonstationary) is rejected, and H_1 : the series are cointegrated (residuals are stationary) is not rejected. In other words, the time-series of Yilan and Hualien are cointegrated. With both stationary and cointegrated check, it can be confirmed that the regression equation of Yilan and Hualien (Equation 5) is not a spurious one.

The regression result of Yilan and Hualien can be expressed as follows:

$$Yilan = 0.2151 Hualien \quad (5)$$

The regression equation shown above means that a unit increase of earthquake in Hualien may result in an increase of 0.2151 unit of earthquake in Yilan.

4.2 Cointegration Test of Taitung and Hualien

The Augmented Dickey-Fuller (ADF) test is still used to check the cointegration test of the residuals of the regression between Hualien and Taitung, but with different critical values (Hill, Griffiths, and Lim). The results are shown in Table 7.

Table 7: Critical values of the cointegration test

$\tau(t)$ Test statistic	1% Critical value	5% Critical value	10% Critical value
-13.31	-3.39	-2.76	-2.45
MacKinnon approximate p -value for $\tau(t) = 0.0000$			

From the above table, one finds the $\tau(t)$ test statistic $-13.31 < -2.76$ (5% critical value), the hypothesis test H_0 : the series are not cointegrated (residuals are nonstationary) is rejected, and H_1 : the series are cointegrated (residuals are stationary) is not rejected. In other words, the series of Taitung and Hualien are cointegrated. With both stationary and cointegrated check, it can be confirmed that the regression equation (Equation 6) is not a spurious one.

$$\text{Taitung} = 0.1692 \text{ Hualien} \quad (6)$$

The regression equation shown above means that a unit increase of earthquake in Hualien may result in an increase of 0.1692 unit of earthquake in Taitung.

5. Autoregressive Conditional Heteroskedastic (ARCH) of the Time Series of Yilan, Hualien and Taitung

Heteroskedasticity exists when the variances for all observations are different. Conversely, when the variances are identical, the homoskedasticity exists. Variable regresses on its own lag or lags is “autoregressive.” Nobel Prize winner Robert Engle’s original work (Engle) on autoregressive conditional heteroskedastic (ARCH) model was concerned with the volatility of inflation (Hill, Griffiths, and Lim). However, the author applies that model and works on the time series of the monthly number of earthquakes in Yilan, Hualien and Taitung. If the heteroskedasticity exists, the range of confidence interval obtained from the standard error of samples should also be modified. The ARCH model has become a popular one because it is useful for modeling volatility and especially true when variances change over time (Hill, Griffiths, and Lim). The earthquake data are volatile, and ARCH may be suitable to trace the variances of such kinds of time series.

5.1 ARCH Test of Yilan

A Lagrange Multiplier (LM) test can be used to test for the presence of ARCH effects. To perform this test, one has to first test the mean equation, save and square the estimated residuals, and perform the statistic test (Hill, Griffiths, and Lim; Salvatore and Reagle). The ARCH test for Yilan is shown in this subsection. The estimated variance \hat{h}_t at time t is shown in Equation 7, and from the t -statistic of the first-order coefficient ($\hat{\alpha}_1=0.324$, $t=5.6$) suggests a significant ARCH(1) coefficient. Hence, the null hypothesis H_0 : no ARCH is rejected and the alternative hypothesis H_1 : ARCH(p) disturbance is accepted.

$$\hat{h}_t = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{e}_{t-1}^2 = 22.68 + 0.324 \hat{e}_{t-1}^2 \quad (7)$$

t (4.01) (5.6)

Where

- \hat{h}_t = estimated variance of residual at time t
- $\hat{\alpha}_0, \hat{\alpha}_1$ = coefficients to be determined
- \hat{e}_{t-1} = error term at time $t-1$

The results of the ARCH(1) model for Yilan are shown in Figure 14. From the figure, it is obviously for one to observe the volatility of variances in the time series of the monthly number of earthquakes in Yilan.

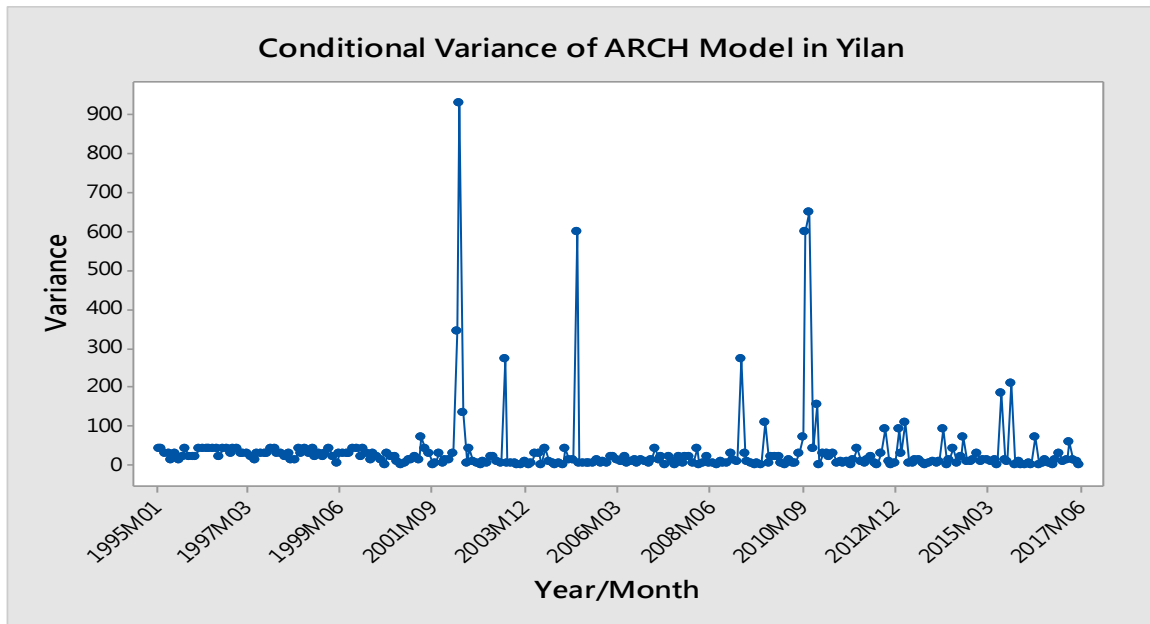


Figure 14: Variance of autoregressive conditional heteroskedasticity (ARCH) model for Yilan

5.2 ARCH Test of Hualien

The ARCH test of Hualien can be expressed in Equation 8. The *t*-statistic of the first-order coefficient ($\hat{\alpha}_1 = 0.250$, $t = 4.22$) suggests a significant ARCH(1) coefficient. One of the requirements for the ARCH model is that $\hat{\alpha}_0 > 0$ and $\hat{\alpha}_1 > 0$ to make sure of the positive variance. From Equation 8, such requirements are satisfied. Hence, the null hypothesis H_0 : no ARCH is rejected and the alternative hypothesis H_1 : ARCH(*p*) disturbance is accepted.

$$\hat{h}_t = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{e}_{t-1}^2 = 293.058 + 0.250 \hat{e}_{t-1}^2 \quad (8)$$

t (3.57) (4.22)

Where

\hat{h}_t = estimated variance of residual at time *t*

$\hat{\alpha}_0, \hat{\alpha}_1$ = coefficients to be determined

\hat{e}_{t-1} = error term at time *t-1*

The results of the ARCH(1) model are shown in Figure 15. From the figure, it is obviously for one to observe the volatility of variances in the time series of the monthly number of earthquakes in Hualien.

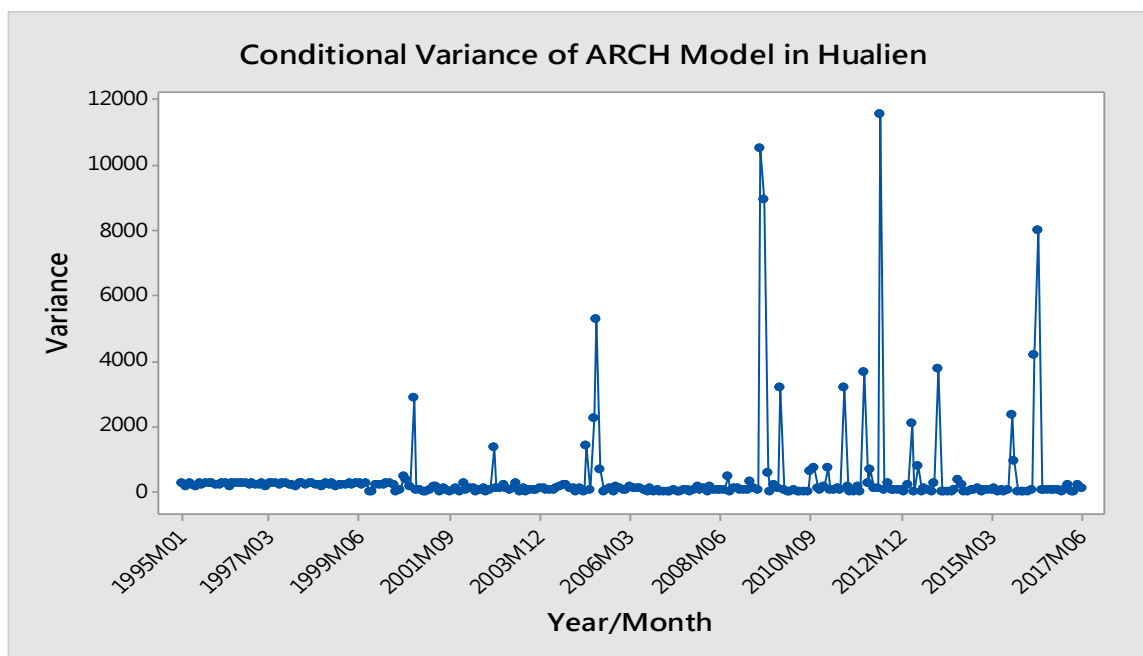


Figure15: Variances of autoregressive conditional heteroskedasticity (ARCH) model for Hualien

5.3 ARCH Test of Taitung

The ARCH test for Taitung is the same as the previous subsection of Hualien, only the analysis results are shown in this subsection. The estimated variance \hat{h}_t at time t is shown in Equation 9.

$$\hat{h}_t = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{e}_{t-1}^2 = 47.587 - 0.013236 \hat{e}_{t-1}^2 \quad (9)$$

t (2.65) (-0.22)

Where

\hat{h}_t = estimated variance of residual at time t

$\hat{\alpha}_0, \hat{\alpha}_1$ = coefficients to be determined

\hat{e}_{t-1} = error term at time $t-1$

The results of the ARCH(1) model for Taitung are shown in Figure 6. The p -value = 0.828 > α (0.05), the H_0 : no ARCH cannot be rejected. Therefore, the variance of monthly number of earthquakes in Taitung is stationary and has no ARCH effect

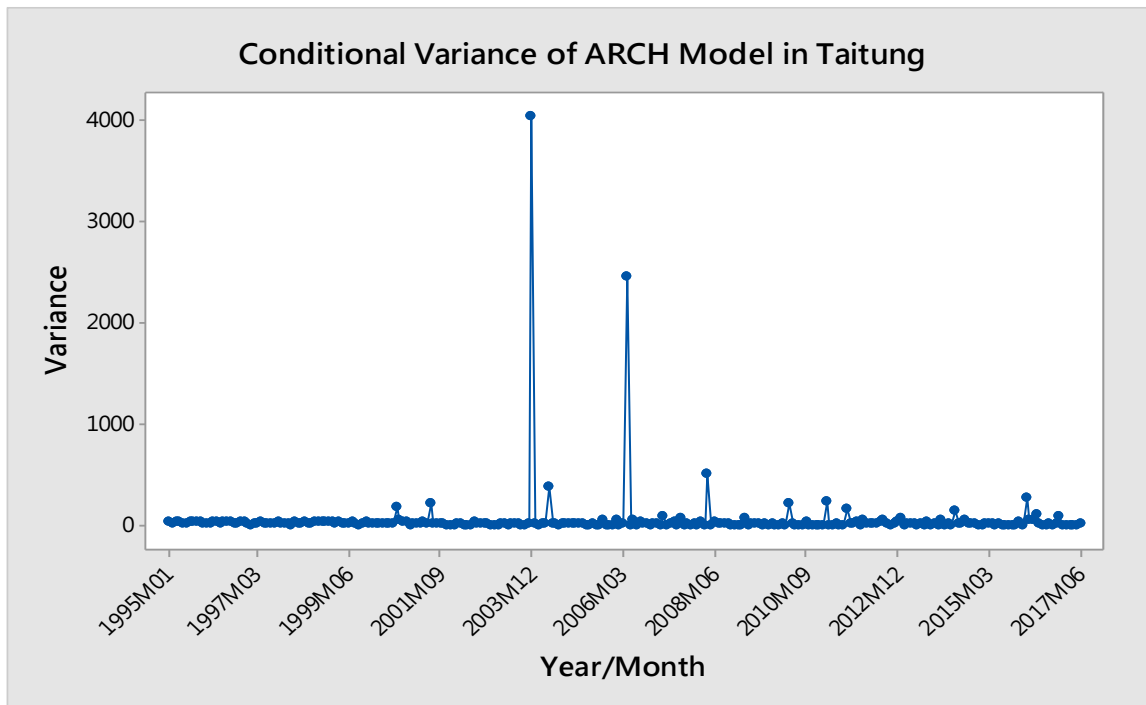


Figure 16: Variance of autoregressive conditional heteroskedasticity (ARCH) model for Taitung

6. Conclusions

After the characteristics of monthly number of earthquakes in Yilan, Hualien and Taitung are analyzed, the following conclusions can be obtained:

- (1) The monthly number of earthquakes from January 1995 to June 2017 in Yilan, Hualien, and Taitung exhibit certain pattern. ARIMA(3,0,0) model is a suitable pattern for Yilan. ARIMA(2,0,1) and ARIMA(1,0,1) models well fit Hualien and Taitung respectively.
- (2) The unit-root test Augmented Dickey-Fuller (ADF) is used to test the stationarity of the time series of the monthly number of earthquakes in Yilan, Hualien, and Taitung. The hypothesis test $H_0 : \gamma = 0$ (nonstationary) is rejected, and $H_1 : \gamma < 0$ (stationary) is not rejected. In other words, the time series of the monthly number of earthquakes per month in all three areas are stationary.
- (3) The cointegrating relationship between the time series of the monthly number of earthquakes from January 1995 to June 2017 for Yilan -Hualien and Taitung-Hualien exist. Hence, the following regression relationships are not spurious.

$$Yilan = 0.2147 Hualien$$

$$Taitung = 0.1692 Hualien$$

- (4) The regression equation shown above means that a unit increase of earthquake in Hualien may result in an increase of 0.2147 unit earthquake in Yilan, and 0.1692 unit earthquake in Taitung.
- (5) From the result of the ARCH(1) model, it is obvious that the volatility of variances in the time series of the monthly number of earthquakes exists in Yilan and Hualien. In other words, variances of the Yilan and Hualien time series show the effect of heteroskedasticity. However, the time series of the monthly number of earthquakes in Taitung does not show ARCH(1) effect, and it is homoskedastic.

7. References

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8. Appendix

8.1 Appendix A: ARIMA(p,q,r) Forecasting Procedures

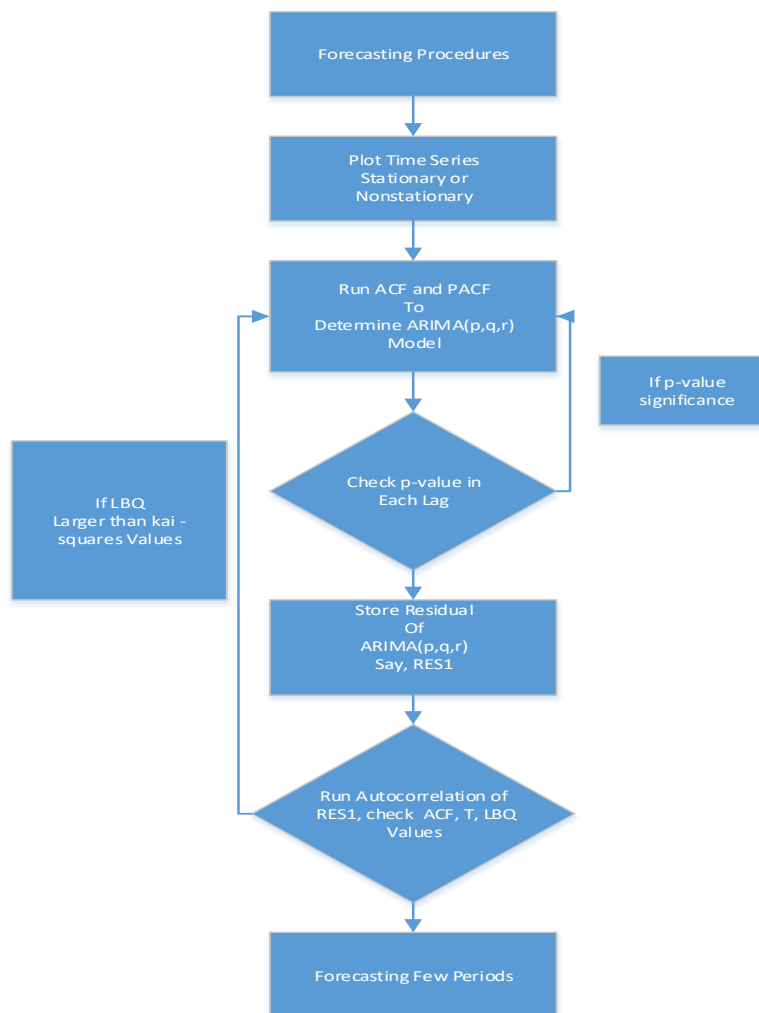


Figure A1: ARIMA(p,q,r) forecasting procedures