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# ANALYSIS OF COMBINING AND REGULARIZATION OPERATIONS ON GEOMETRICAL IMAGES OF AUTOMATA MODELS OF DISCRETE SYSTEMS

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**Abstract**

In paper are spend analysis of two classes of operations on geometric images of automata models of discrete dynamical systems. The operations of combining and regularization of geometrical images of automaton mappings are investigated. As a means of regularization of the partially defined automaton mappings are used both the classical interpolation methods of the numerical graphs and their new modifications. Various operations of combining of geometrical images are investigated. For the selected class of combining operations is made an analysis of the dependence of the number of states of the automaton, the geometric image of which is obtained as a result of the juxtaposition of geometric images of basic automatons, from the powers of the sets of states of basic automatons. The effectiveness of various methods of interpolation is investigated for regularization of partially defined geometric images of automatons.

**Key Words**: discrete deterministic dynamical system, mathematical model, automaton, geometric image of automata mapping, interpolation, combination of geometric images of automatons.

## 1 . Introduction.

At exploitation of modern complex man-machine systems (air transport, space systems, nuclear, wind and hydroelectric power stations, railways and motor roads, large industrial enterprises for various purposes, gas and oil networks, oil pipelines, telecommunications networks, distributed computing systems), there may be emergencies (events) in the event of a negative development of which (in the absence of parrying a defect or in case of parrying non-rules the recognized defect, etc.) can lead to an accident or catastrophe, the consequences of which, in turn, can substantially block the economic and other effects from the use of such systems. In this regard, the problem of ensuring and maintaining the safety of functioning of modern complex man-machine systems is very relevant and represents the subject of research of a huge number of teams of both domestic and foreign scientists and specialists.

The problem of ensuring and maintaining the security of the functioning of complex systems includes the tasks of control and diagnosing of systems. One of the main difficulties that do not allow the effective dissemination of all the variety of methods of technical diagnosis developed for low-dimensional systems is the problem of constructing mathematical models of systems (both complex technical systems and complex man-machine systems).

The fundamental impossibility of constructing a complete, accurate and compact model of complex man-machine systems (as well as complex technical systems) is due to the enormous dimension of such systems, the complex structure, the large number of connections between components and subsystems, the presence of uncertainties in functioning, etc. In view of the above features of complex man-machine systems (CMMS) is being conducted research on the development of methods of constructing a model of the CMMS in a whole (taking into account the heterogeneity of processes in the CMMS, changes in the CMMS in time) on the basis of a limited set of fully or partially defined actual processes of the system operation, and also for constructing a mathematical model of the system in a whole, based on models of individual components of the system.

The novelty of the paper is that it is proposed to develop methods of regularization for partially set laws of functioning of the automatons, presented exactly in the geometrical images and exactly with use of interpolation methods. Such methods allow to take into account the specificity of automaton models of complex systems by using: the base points, which second coordinates are received by sections of geometrical images by the straight lines parallel to an axis of abscissas; the base points of interpolation allocated with the first elements of some tops of geometrical images (for this, autonomous subautomatons are used).

## 2. Materials and Methods: Geometrical form of mathematical models of complex discrete dynamical systems.

It is known, that the apparatus of continuous numerical mathematics effectively uses infinite sets. In this connection Tverdokhlebov V. A. was developed the new approach to construction of models of difficult systems and methods of the analysis of such models, which are stated in works [1, 2]. A developed principle is placing of discrete structures on continuous geometrical curves, as a rule, set analytically. For this purpose instead of next-state function and output function of automaton is considered a automaton mapping, i.e. symbolical mathematical structures of a kind (input sequence, output sequence).

These structures rely points, which after introduction of linear ordered on set of all input sequences and on set of all output sequences, form the symbolical graphic. Numberings of input and output sequences on the basis of the entered linear orders transform the symbolical graphic into the numerical curve. Possibility of placing of discrete automaton mapping on continuous (numerical) geometrical curves, that allows to work with laws of functioning of automatons with the methods, developed in the continuous mathematics for geometrical curves turns out.

Geometrical image $\gamma_s$ of laws of functioning (see works [1,2]) (next-state function $\delta$: $S\times X\rightarrow S$ and functions of outputs $\lambda$: $S\times X\rightarrow Y$) of initial finite determined automaton $A_s = (S, X, Y, \delta, \lambda, s)$ with sets of states $S$, input signals $X$ and output signals $Y$ it is defined on the basis of introduction of a linear order $\omega$ in automata mapping $\rho'_s = \bigcup_{p\in X^*}\{(p,\lambda(s,p))\}$, where $\lambda(s,p)=\lambda(\delta(s,p'),x)$, at $p=p'x$. Automaton mapping $\rho_s$ (set of pairs) is ordered by linear order $\omega$, defined on the basis of an order $\omega_1$ on $X^*$ and set by following rules:

Rule 1. On set $X$ some linear order $\omega_1$ (which we will designate $\prec_1$) is entered

Rule 2. An order $\omega_1$ on $X$ we will extend to a linear order on set $X^*$, believing, that

- For any words $p_1, p_2 \in X^*$ with unequal length ( $|p_1| \neq |p_2|$) $|p_1| < |p_2| \rightarrow p_1 \prec_1 p_2$;

- For any words $p_1, p_2 \in X^*$ for which $|p_1| = |p_2|$ and $p_1 \neq p_2$, their relation in the order of $\omega_1$ repeats the relation of the incoincident letters of words, nearest at the left in $p_1$ and $p_2$. The order $\omega_2$ on set of words $Y^*$ is similarly defined.

After introduction on set $X^*$ of a linear order $\omega_1$, we receive linearly ordered set $\rho_s = (\rho_s', \omega_1')$, where $\omega_1'$ - an order on $\rho_s'$, induced rather $\omega_1$ on $X^*$.

Having a linear order $\omega_2$, defined on set $Y$ and having placed set of points $\rho_s$ in system of coordinates $D_1$ with an axis of abscisses $(X^*, \omega_1)$ and an axis of ordinates $(Y, \omega_2)$, we receive a geometrical image $\gamma_s$ of laws of functioning of initial finite determined automaton $A_s = (S, X, Y, \delta, \lambda, s)$. Linear orderes $\omega_1$ and $\omega_2$ allow to replace elements of sets $X^*$ and $Y$ by their numbers $r_1(p)$ and $r_2(p)$ on these orders.

**3. Synthesis of automatons by geometric curves.** Proposed and developed by Tverdokhlebov V.A. the apparatus of geometric images of automatons allows to consider geometric curves with automaton interpretation. The use of geometric images introduces methods of automata theory into the analysis of geometric curves.

For an initial finite deterministic automaton $(A, s_0)$, where $A = (S, X, Y, \delta, \lambda)$ and all five components of the automaton are required to determine. For the set of states of the automaton we use the notation: $S = \{s_p\}_{p \in X^*}$, where $s_{px} = \delta(s_p, x)$, $x \in X$, $p \in X^*$, $s_0 = s_\varepsilon$. This allows to introduce and use the standard definition of the next-state function δ with the possible subsequent minimization of the automaton with respect to the number of states and the corresponding correction of the function δ.

The method of synthesis the laws of the functioning of a discrete deterministic automaton by a given geometric figure proposed and developed by V.A. Tverdokhlebov in [1] establishes a one-to-one correspondence between the functions δ and λ and a geometric figure based on the selected and fixed traversal of the line and points on the curve. In fact, in the method the curve line is represented by a sequence of points, the choice of which is not unique without additional conditions. In this paper, we research the properties of the laws of the functioning of discrete deterministic dynamical systems represented in the form of geometric curves.

Based on the method proposed by V.A. Tverdokhlebov in [1], are constructed for each of the analyzed curves 12 automatons (for different values of the power of the input alphabet of the automaton, | X | = 2, 5, 10, 25). The set of geometric curves extracted from the bank [5] consists of 50 geometric curves, so the total number of automatons is 600. An important way is to extend the function of the transitions δ of the automaton. Cyclic regularization, regularization to the initial state, generation of the state in a pseudo-random manner (from a set of possible states) is possible. In a case, when $\frac{k}{|X|} \neq \left[\frac{k}{|X|}\right]$, where |X| - the power of the input alphabet of the automaton, and k - is the number of points on the curve (along which the laws of the functioning of the automaton are constructed), an regularization is required for the output function λ.

In this paper, the definition of the next-state function is carried out by all these methods, and the value of the input alphabet power and the number of points are chosen in such a way that $\frac{k}{|X|} = \left[\frac{k}{|X|}\right]$, therefore, the definition of the function λ is not required.

As an example, in table 1 we give the definition of an automaton (for | X | = 10) constructed by the Fibonacci spiral (in the curve approximation by 30 points) when the next-state function is cycled.

The selection of classes of equivalent states showed that for all 600 automatons constructed by 50 geometric curves (when 30 points are chosen on curves), the number of equivalence classes coincides with the number of states of the automaton, i.e. automatons are already minimal in the number of states.

This property is present in all 600 automatons constructed with all the methods of regularization of next-state function of the automaton: when the next-state function is cycled, when it is extended to the initial state, when it is redefined using a random number generator (the state is randomly selected from a variety of possible states).

*Table 1 - Next-state and output functions of automaton constructed by Fibonacci spiral*

| $\delta$ | $s_0$ | $s_1$ | $s_2$ |
|---|---|---|---|
| $x_1$ | $s_1$ | $s_0$ | $s_1$ |
| $x_2$ | $s_2$ | $s_1$ | $s_2$ |
| $x_3$ | $s_0$ | $s_2$ | $s_0$ |
| $x_4$ | $s_1$ | $s_0$ | $s_1$ |
| $x_5$ | $s_2$ | $s_1$ | $s_2$ |
| $x_6$ | $s_0$ | $s_2$ | $s_0$ |
| $x_7$ | $s_1$ | $s_0$ | $s_1$ |
| $x_8$ | $s_2$ | $s_1$ | $s_2$ |
| $x_9$ | $s_0$ | $s_2$ | $s_0$ |
| $x_{10}$ | $s_1$ | $s_0$ | $s_1$ |

| $\lambda$ | $s_0$ | $s_1$ | $s_2$ |
|---|---|---|---|
| $x_1$ | $y_5$ | $y_1$ | $y_{12}$ |
| $x_2$ | $y_4$ | $y_0$ | $y_{13}$ |
| $x_3$ | $y_3$ | $y_1$ | $y_{14}$ |
| $x_4$ | $y_5$ | $y_2$ | $y_{15}$ |
| $x_5$ | $y_6$ | $y_3$ | $y_{16}$ |
| $x_6$ | $y_7$ | $y_7$ | $y_{17}$ |
| $x_7$ | $y_6$ | $y_8$ | $y_{18}$ |
| $x_8$ | $y_5$ | $y_9$ | $y_{18}$ |
| $x_9$ | $y_3$ | $y_{10}$ | $y_{18}$ |
| $x_{10}$ | $y_2$ | $y_{11}$ | $y_{17}$ |

**4. Analysis of the operations of combining geometric images of automatons.** In this part of the paper are investigated the operations of combining of geometric images of automatons. Are realized the construction and analysis of automatons, the geometric images of which are obtained by combining from the basic set of curves.

We consider 50 2D curves from the bank [5] and two operations of combining of geometric curves (addition and subtraction). The result of the addition of the curves $y_1=f_1(x)$ and $y_2=f_2(x)$

is the curve $y=f_1(x)+f_2(x)$, subtraction - $y=f_1(x)-f_2(x)$. Investigated the dependence of the number of states in minimal automata constructed from the base curves and from the curves obtained as a result of combining of the basic ones with the number of input signals of the automaton, from the method of regularization of next-state function, and the type of the operation of combining the curves. As a result of overlapping of 50 basic geometric curves, 2500 curves are obtained with the addition operation and 2500 curves using the subtraction operation.

The synthesis of the laws of the functioning of automatons by curves is carried out with three methods of regularization of next-state function of the automaton (cyclic, to the initial state, random generation of the state), and the following number of input signals of the automaton: $|X| = 2, 5, 10, 25, 50$. As an example, on fig. 1 are shown 2 curves $y_1=f_1(x)$ and $y_2=f_2(x)$ from the basic set of curves, and the curve $y = f_1(x)+ f_2(x)$, obtained as a result of addition. The analytical specification of these curves has the following form: $y_1=\frac{|x|^5+15}{|x|^2+1}$, $y_2 = e^x$. The automatons constructed on these curves for the selection of points on the curves 20, cyclic regularization of next-state function of the automatons and $|X| = 5$ are given in tables 2-4. In the above example, the number of states after minimization for the automata $A_1$ and $A_2$ constructed from the curves $y_1=\frac{|x|^5+15}{|x|^2+1}$ и $y_2 = e^x$, and also from the automaton B constructed from the curve $y=\frac{|x|^5+15}{|x|^2+1}+e^x$, does not change, i.e. automata are already minimal in number of states.

*Table 2 - Next-state and output functions of automaton $A_1$ (with $|X| = 5$ ) constructed by geometrical curve $y_1$*

| $\delta$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| $x_1$ | $s_1$ | $s_2$ | $s_3$ | $s_0$ |
| $x_2$ | $s_2$ | $s_3$ | $s_0$ | $s_1$ |
| $x_3$ | $s_3$ | $s_0$ | $s_1$ | $s_2$ |
| $x_4$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
| $x_5$ | $s_1$ | $s_2$ | $s_3$ | $s_0$ |

| $\lambda$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| $x_1$ | $y_6$ | $y_0$ | $y_4$ | $y_1$ |
| $x_2$ | $y_5$ | $y_1$ | $y_3$ | $y_2$ |
| $x_3$ | $y_4$ | $y_2$ | $y_2$ | $y_4$ |
| $x_4$ | $y_2$ | $y_3$ | $y_1$ | $y_5$ |
| $x_5$ | $y_1$ | $y_4$ | $y_0$ | $y_6$ |

It is noted that when used regularization of the next-state function of automaton to the initial state and the value $|X| = 2$, the number of states for the automaton B decreases after minimization, whereas for the automata $A_1$ and $A_2$, the number of states does not decrease. In the case when the power of the input alphabet of the automaton is equal to 10 the number of classes of equivalent states for the automata $A_1$, $A_2$ and B for all used regularization methods to determine the next-state function (cyclic, to the initial state and randomization) coincides with the number of states (i.e., automatons are minimal).

*Table 3 - Next-state and output functions of automaton $A_1$ (with $|X| = 5$ ) constructed by geometrical curve $y_2=e^x$*

| $\delta$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| $x_1$ | $s_1$ | $s_2$ | $s_3$ | $s_0$ |
| $x_2$ | $s_2$ | $s_3$ | $s_0$ | $s_1$ |
| $x_3$ | $s_3$ | $s_0$ | $s_1$ | $s_2$ |
| $x_4$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
| $x_5$ | $s_1$ | $s_2$ | $s_3$ | $s_0$ |

| $\lambda$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| $x_1$ | $y_0$ | $y_5$ | $y_{10}$ | $y_{15}$ |
| $x_2$ | $y_1$ | $y_6$ | $y_{11}$ | $y_{16}$ |
| $x_3$ | $y_2$ | $y_7$ | $y_{12}$ | $y_{17}$ |
| $x_4$ | $y_3$ | $y_8$ | $y_{13}$ | $y_{18}$ |
| $x_5$ | $y_4$ | $y_9$ | $y_{14}$ | $y_{19}$ |

As a result of the construction and analysis of 75,000 automatons with 5000 curves obtained as a result of combining of basic curves, it was noted that the use of both the addition operation and the subtraction operation for the values $|X| = 5, 10, 25, 50$ and the definition functions of automata transitions randomly (using a pseudo-random variable generator) does not reduce the number of states in the automatons.

*Table 4 - Next-state and output functions of automaton B (with $|X| = 5$ ) constructed by geometrical curve* $y = \frac{|x|^5 + 15}{|x|^2 + 1} + e^x$

| $\delta$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| $x_1$ | $s_1$ | $s_2$ | $s_3$ | $s_0$ |
| $x_2$ | $s_2$ | $s_3$ | $s_0$ | $s_1$ |
| $x_3$ | $s_3$ | $s_0$ | $s_1$ | $s_2$ |
| $x_4$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
| $x_5$ | $s_1$ | $s_2$ | $s_3$ | $s_0$ |

| $\lambda$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
|---|---|---|---|---|
| $x_1$ | $y_7$ | $y_1$ | $y_4$ | $y_3$ |
| $x_2$ | $y_6$ | $y_0$ | $y_5$ | $y_4$ |
| $x_3$ | $y_5$ | $y_1$ | $y_4$ | $y_5$ |
| $x_4$ | $y_4$ | $y_2$ | $y_3$ | $y_6$ |
| $x_5$ | $y_2$ | $y_3$ | $y_2$ | $y_7$ |

Using the additional definition (regularization) of next-state function to the initial state and the values $|X| = 2, 5$, it is possible to reduce the number of states after minimization, both for automatons constructed from basic curves and for automatons constructed from the curves obtained as a result of combining and with the addition operation and using the subtraction operation. An analysis of automatons constructed using all these methods of regularization of next-state function and at values $|X| = 10, 25, 50$ showed that after minimization the number of states does not decrease.

Also, the analysis of automatons constructed from curves obtained as a result of combining the basic curves by means of the operation of exponentiation, i.e., the result of combining $y_1=f_1(x)$ and $y_2=f_2(x)$ from the base set of curves is assumed to be curve $y=f_1(x)^{f_2(x)}$.

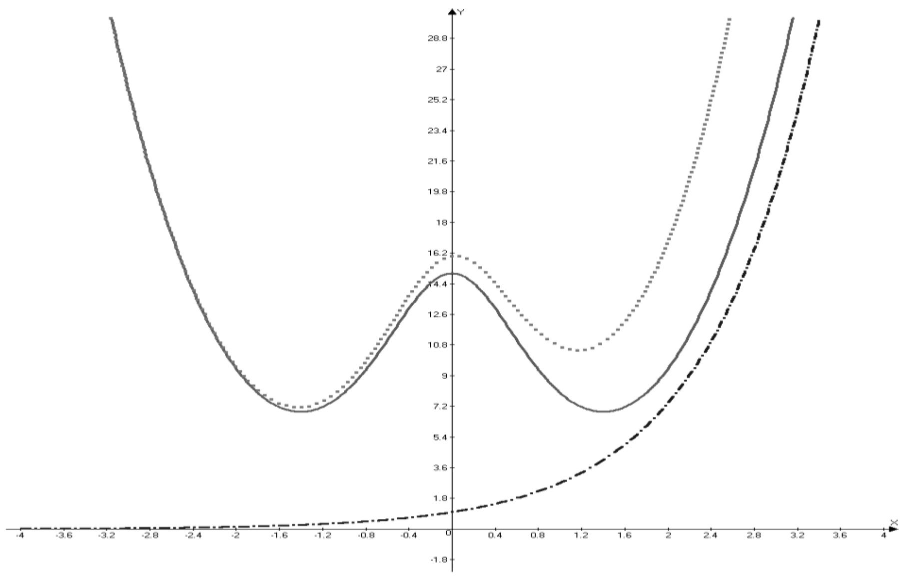


*Figure 1 - Graphics of functions*

——— $y_1=\dfrac{|x|^5+15}{|x|^2+1}$, ............. $y=\dfrac{|x|^5+15}{|x|^2+1}+e^x$.

– · – · – $y_2 = e^x$ ,

As a result of this combination of geometric images, more than 1500 curves have been constructed, according to which the laws of the functioning of more than 18,000 automatons are synthesized, with different ways of regularization) of next-state function of the automaton and various values of the power of the input alphabet. It is noted that for certain combinations of the curves $y_1=f_1(x)$ and $y_2 = f_2(x)$ and $y_2=f_2(x)$ and the values $|X| = 2$ and $|X| = 5$, the number of states can be reduced after minimization of the automaton constructed from the curve obtained as a result of combining the curves $y_1=f_1(x)$ and $y_2=f_2(x)$.

## 5. Development of methods for increasing the completeness and accuracy of automaton models of systems using the interpolation of geometric images of automatons.

In the fundamental papers containing the development of the theory of automatons, the problem of regularization of partially set automatons on the basis of a unified approach is not considered. There are problems in the solution of which the methods used assume completely established laws of automata functioning, and in the initial data these laws are presented only partially.

Fundamental mathematical results on the regularization of partially specified graphics are represented by classical interpolation methods of Newton, Lagrange, Gauss, Bessel, Stirling, spline interpolation methods, least-squares method, nearest-neighbor method, etc. The inapplicability of these methods for partially specified automatons is related to the symbolic form of setting automatons by tables, matrices, graphs, systems of logical equations, and so on. The definition of the laws of automata functioning by numerical structures on the basis of the representation of automatons mappings by numerical graphs [1, 2] allows to use classical interpolation methods in the theory of automatons. In this paper we consider the interpolation of the laws of functioning of automatons on the basis of the application of developed methods to classes of automatons and their subclasses formed by combinations of Post properties for combinational parts of automatons.

In this section are present the results of an analysis of the effectiveness of the application of the classical interpolation methods of Newton, Lagrange, Gauss, and others for regularization the partially defined laws for of functioning of automatons in a subclass of linear (8,2,2)-automatons. The class of (8,2,2) - automatons consists of 18446744073709551616 automatons.

To perform the analysis of the efficiency of the regularization for the whole class of (8,2,2) - automatons, even when considering the initial lengths of geometric images of length 2046 (which corresponds to the functioning of the automaton in words up to length 10 inclusive) and the recovery rate of 1,000,000 partially specified geometric images per second, more than 580,000 years will be required. In addition, the definition of an effective interpolation method for regularization the laws governing the operation of automatons is not possible without taking into account the specific of laws of the operation of automatons. In view of this, the problem of analyzing the effectiveness of the application of classical interpolation methods in order to regularization the laws of the operation of automatons in particular subclasses is topical.

The selection for the analysis of subclasses of automata is carried out on the basis of a new classification of finite determinate automata, proposed in [1]. It is assumed that the automaton is defined as follows: $A=(\{0,1\}^3,\{0,1\}, \{0,1\}, <\delta_1, \delta_2, \delta_3>, \lambda)$, where functions $\delta_1, \delta_2, \delta_3$ and $\lambda$ - Boolean functions of a kind $\delta_i:\{0,1\}^3\times\{0,1\}\rightarrow\{0,1\}$, $\lambda:\{0,1\}^3\times\{0,1\}\rightarrow\{0,1\}$. Using the classification of automata proposed in [1] based on the use of the property of five Post classes (monotonic, affine, self-dual, truth-preserving, falsity-preserving, see [1]) for characteristics of the next-state functions $<\delta_1, \delta_2, \delta_3>$ and the outputs function $\lambda$, as well as combinations of these properties , from the (8,2,2) - automatons class, 15 non-empty subclasses of automata were singled out.

To denote the class of automatons whose next-state and output functions belong to the class of functions of the algebra of logic $K_{ij\omega uv} = K_a \cap K_b \cap K_c \cap K_d \cap K_e$, where $i = \begin{cases} 0, a = \bar{0} \\ 1, a = 0 \end{cases}$, $j = \begin{cases} 0, b = \bar{1} \\ 1, b = 1 \end{cases}$ , $\omega = \begin{cases} 0, c = \bar{L} \\ 1, c = L \end{cases}$, $u = \begin{cases} 0, d = \bar{S} \\ 1, d = S \end{cases}$ , $v = \begin{cases} 0, e = \bar{M} \\ 1, e = M \end{cases}$ , we will use the letter H with five lower indices. For example: $H_{01100}$ - a class of automatons whose next-state and output functions belong to the class of functions of the algebra of logic $K_{01100} = K_{\bar{0}} \cap K_1 \cap K_L \cap K_{\bar{S}} \cap K_{\bar{M}}$.

Information on the power of each considered of the 15 subclasses of the class of functions of the algebra of logic of 4 variables is given in table 5, and in table 6 - information about subclasses of the (8,2,2) - automatons class and properties of the next-state and output functions.

*Table 5 - non-empty subclasses of class of Boolean functions from 4 variables*

| Automaton subclass number | Property of subclass | Number of functions in subclass |
|---|---|---|
| 1 | $K_{00000}$ | 16256 |
| 2 | $K_{00010}$ | 120 |
| 3 | $K_{00110}$ | 8 |
| 4 | $K_{01000}$ | 16376 |
| 5 | $K_{01100}$ | 7 |
| 6 | $K_{01101}$ | 1 |
| 7 | $K_{10000}$ | 16376 |
| 8 | $K_{10100}$ | 7 |
| 9 | $K_{10101}$ | 1 |
| 10 | $K_{11000}$ | 16102 |
| 11 | $K_{11001}$ | 154 |
| 12 | $K_{11010}$ | 112 |
| 13 | $K_{11011}$ | 8 |
| 14 | $K_{11110}$ | 4 |
| 15 | $K_{11111}$ | 4 |

In this paper we analyze the effectiveness of using of classical interpolation methods to partially defined geometric images of automata from subclasses $H_{00110}$, $H_{01100}$, $H_{10100}$, $H_{11011}$, $H_{11110}$ and $H_{11111}$ of (8,2,2) -automatons class. A subclass of linear (8.2,2)-automatons (8388608 initial automata) is also considered, class of automatons defined as follows: $A=(\{0,1\}^3,\{0,1\}, \{0,1\}, <\delta_1, \delta_2, \delta_3>, \lambda)$, where the functions $\delta_1$, $\delta_2$, $\delta_3$ and $\lambda$ are linear Boolean functions of 4 variables of a kind $\delta_i:\{0,1\}^3\times\{0,1\}\rightarrow\{0,1\}$, $\lambda:\{0,1\}^3\times\{0,1\}\rightarrow\{0,1\}$.

The following theorem reflects the results of the analysis of the effectiveness of the application of the Newton and Lagrange interpolation methods to partially-given geometric images of autonomous subautomatons of the class of linear (8,2,2) -automatons.

Theorem 1. Let $G_d$, where $d \in \{30, 62, 126, 254\}$ - set of geometrical images of length d of initial discrete deterministic automatons of a kind $A=(\{s_1,s_2,\ldots,s_8\},\{x_1,x_2\},\{y_1, y_2\}, \delta, \lambda, s_0 \in\{s_1, s_2,..., s_8\})$ from a class of linear (8,2,2)-automatons, and $G_d^1$ $(G_d^0)$ - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$ . Then :

- at d = 30 for 5578433 automatons $n_d^N > n_d^L$, for 607351 automatons $n_d^N < n_d^L$, for 2202824 automatons $n_d^N = n_d^L$;

- at d = 62 for 4494945 automatons $n_d^N > n_d^L$, for 744490 automatons $n_d^N < n_d^L$, for 3149173 automatons $n_d^N = n_d^L$;

- at d = 126 for 1426944 automatons $n_d^N > n_d^L$, for 1483776 automatons $n_d^N < n_d^L$, for 5477888 automatons $n_d^N = n_d^L$;

- at d = 254 for 1492352 automatons $n_d^N > n_d^L$, for 1402240 automatons $n_d^N < n_d^L$, for 5494016 automatons $n_d^N = n_d^L$.

*Table 6 -15 nonempty subclasses of (8,2,2)-automatons class*

| Automaton subclass number | The class to which the next-state and output functions belong | Number of automatons in subclass | Number of initial automatons |
|---|---|---|---|
| 1 | $H_{00000}$ | 69832045332791296 | 558656362662330368 |
| 2 | $H_{00010}$ | 207360000 | 1658880000 |
| 3 | $H_{00110}$ | 4096 | 32768 |
| 4 | $H_{01000}$ | 71916959595237376 | 575335676761899008 |
| 5 | $H_{01100}$ | 2401 | 19208 |
| 6 | $H_{01101}$ | 1 | 8 |
| 7 | $H_{10000}$ | 71916959595237376 | 575335676761899008 |
| 8 | $H_{10100}$ | 2401 | 19208 |
| 9 | $H_{10101}$ | 1 | 8 |
| 10 | $H_{11000}$ | 67223216569555216 | 537785732556441728 |
| 11 | $H_{11001}$ | 562448656 | 4499589248 |
| 12 | $H_{11010}$ | 157351936 | 1258815488 |
| 13 | $H_{11011}$ | 4096 | 32768 |
| 14 | $H_{11110}$ | 256 | 2048 |
| 15 | $H_{11111}$ | 256 | 2048 |

Figure 2 shows a schematic partitioning of the class of linear (8,2,2)-automatons into subclasses $M_1^d, M_2^d, M_3^d$, $d \in \{30, 62, 126, 254\}$, where $M_1^d$ ($M_2^d$, $M_3^d$) for concrete value $d$, respectively, subclasses of automatons for which the number of correctly restored points by Newton's method is strictly greater than the number (strictly less than a number, equal to the number) of correctly restored points by the Lagrange method.

The following Theorems 2-9 reflect the results of the analysis of the effectiveness of Newton's and Lagrange's methods in subclasses $H_{00110}$, $H_{01100}$, $H_{01101}$, $H_{10100}$, $H_{10101}$, $H_{11011}$, $H_{11110}$ и $H_{11111}$ of (8,2,2)-automatons class**.**

Theorem 2. Let $G_d$, where $d \in \{30, 62, 126, 254\}$, - set of geometrical images of length $d$ of initial discrete deterministic automatons of a kind A=({$s_1,s_2,\ldots,s_8$},{$x_1,x_2$},{$y_1$, $y_2$},δ, λ,$s_0 \in \{s_1, s_2, \ldots, s_8\}$) from subclass $H_{00110}$ of (8,2,2)-automatons class and $G_d^1$ ($G_d^0$) - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$. Then:

- at d = 30 for 23281 automatons $n_d^N > n_d^L$, for 1320 automatons $n_d^N < n_d^L$, for 8167 automatons $n_d^N = n_d^L$;

- at d = 62 for 17456 automatons $n_d^N > n_d^L$, for 1524 automatons $n_d^N < n_d^L$, for 13788 automatons $n_d^N = n_d^L$;

- at d = 126 for 3168 automatons $n_d^N > n_d^L$, for 4672 automatons $n_d^N < n_d^L$, for 24928 automatons $n_d^N = n_d^L$;

- at d = 254 for 4360 automatons $n_d^N > n_d^L$, for 3240 automatons $n_d^N < n_d^L$, for 25168 automatons $n_d^N = n_d^L$.

Theorem 3. Let $G_d$, where $d \in \{30, 62, 126, 254\}$, - set of geometrical images of length $d$ of initial discrete deterministic automatons of a kind A=({$s_1,s_2,\ldots,s_8$},{$x_1,x_2$},{$y_1$, $y_2$},δ, λ,$s_0 \in \{s_1, s_2, \ldots, s_8\}$) from subclass $H_{01100}$ of (8,2,2)-automatons class and $G_d^1$ ($G_d^0$) - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$. Then:

- at d = 30 for 12870 automatons $n_d^N > n_d^L$, for 1300 automatons $n_d^N < n_d^L$, for 5038 automatons $n_d^N = n_d^L$;

- at d = 62 for 10214 automatons $n_d^N > n_d^L$, for 2092 automatons $n_d^N < n_d^L$, for 6902 automatons $n_d^N = n_d^L$;

- at d = 126 for 4420 automatons $n_d^N > n_d^L$, for 2712 automatons $n_d^N < n_d^L$, for 12076 automatons $n_d^N = n_d^L$;

- at d = 254 for 4606 automatons $n_d^N > n_d^L$, for 2610 automatons $n_d^N < n_d^L$, for 11992 automatons $n_d^N = n_d^L$.

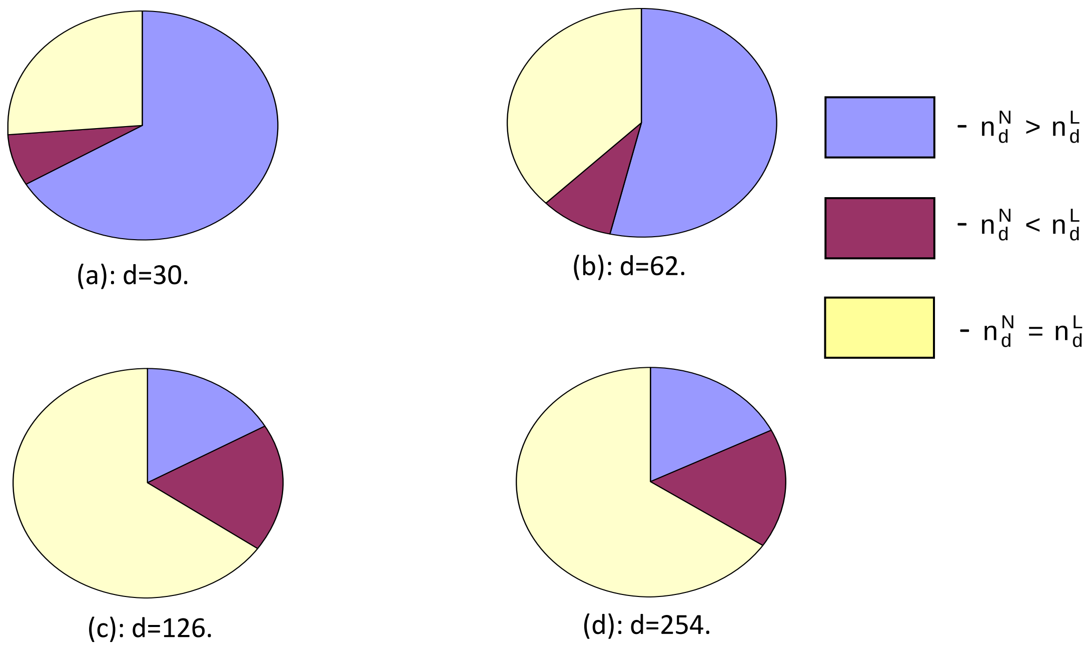


*Figure 2 - schematic partitioning of the class of linear (8,2,2) automatons into subclasses* $M_1^d$, $M_2^d$, $M_3^d$, $d \in \{30, 62, 126, 254\}$, *on the basis of analysis of effectiveness of the application of the Newton and Lagrange interpolation methods in relation to partially defined geometric images*

Theorem 4. Let $G_d$, where $d \in \{30, 62, 126, 254\}$, - set of geometrical images of length $d$ of initial discrete deterministic automatons of a kind $A=(\{s_1,s_2,\ldots,s_8\},\{x_1,x_2\},\{y_1, y_2\},\delta, \lambda, s_0 \in \{s_1, s_2, \ldots, s_8\})$ from subclass $H_{01101}$ of (8,2,2)-automatons class and $G_d^1$ $(G_d^0)$ - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$. Then for any $d \in \{30, 62, 126, 254\}$ for all automatons from this subclass inequality $n_d^N > n_d^L$ is true.

Theorem 5. Let $G_d$, where $d \in \{30, 62, 126, 254\}$, - set of geometrical images of length $d$ of initial discrete deterministic automatons of a kind $A=(\{s_1,s_2,\ldots,s_8\},\{x_1,x_2\},\{y_1, y_2\},\delta, \lambda, s_0 \in \{s_1, s_2, \ldots, s_8\})$ from subclass $H_{10100}$ of (8,2,2)-automatons class and $G_d^1$ $(G_d^0)$ - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$. Then:

- at d = 30 for 11567 automatons $n_d^N > n_d^L$, for 2380 automatons $n_d^N < n_d^L$, for 5261 automatons $n_d^N = n_d^L$;

- at d = 62 for 9715 automatons $n_d^N > n_d^L$, for 3184 automatons $n_d^N < n_d^L$, for 6309 automatons $n_d^N = n_d^L$;

- at d = 126 for 2916 automatons $n_d^N > n_d^L$, for 4216 automatons $n_d^N < n_d^L$, for 12076 automatons $n_d^N = n_d^L$;

- at d = 254 for 2682 automatons $n_d^N > n_d^L$, for 4342 automatons $n_d^N < n_d^L$, for 12184 automatons $n_d^N = n_d^L$.

<u>Theorem 6.</u> Let $G_d$, where $d \in \{30, 62, 126, 254\}$, - set of geometrical images of length d of initial discrete deterministic automatons of a kind $A=(\{s_1,s_2,\dots,s_8\},\{x_1,x_2\},\{y_1, y_2\},\delta, \lambda, s_0 \in \{s_1, s_2, \dots, s_8\})$ from subclass $H_{10101}$ of (8,2,2)-automatons class and $G_d^1$ $(G_d^0)$ - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$. Then for all automatons from this subclass equality $n_d^N = n_d^L$ is true.

<u>Theorem 7.</u> Let $G_d$, where $d \in \{30, 62, 126, 254\}$, - set of geometrical images of length d of initial discrete deterministic automatons of a kind $A=(\{s_1,s_2,\dots,s_8\},\{x_1,x_2\},\{y_1, y_2\},\delta, \lambda, s_0 \in \{s_1, s_2, \dots, s_8\})$ from subclass $H_{11011}$ of (8,2,2)-automatons class and $G_d^1$ $(G_d^0)$ - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$. Then:

- at d = 30 for 17538 automatons $n_d^N > n_d^L$, for 15230 automatons $n_d^N = n_d^L$;

- at d = 62 for 17530 automatons $n_d^N > n_d^L$, for 88 automatons $n_d^N < n_d^L$, for 15150 automatons $n_d^N = n_d^L$;

- at $d \in \{126, 254\}$ for 9262 automatons $n_d^N > n_d^L$, for 23506 automatons $n_d^N = n_d^L$.

<u>Theorem 8.</u> Let $G_d$, where $d \in \{30, 62, 126, 254\}$, - set of geometrical images of length d of initial discrete deterministic automatons of a kind $A=(\{s_1,s_2,\dots,s_8\},\{x_1,x_2\},\{y_1, y_2\},\delta, \lambda, s_0 \in \{s_1, s_2, \dots, s_8\})$ from subclass $H_{11110}$ of (8,2,2)-automatons class and $G_d^1$ $(G_d^0)$ - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$. Then:

- at d = 30 for 1292 automatons $n_d^N > n_d^L$, for 64 automatons $n_d^N < n_d^L$, for 692 automatons $n_d^N = n_d^L$;

- at d = 62 for 1058 automatons $n_d^N > n_d^L$, for 94 automatons $n_d^N < n_d^L$, for 896 automatons $n_d^N = n_d^L$;

- at d = 126 for 202 automatons $n_d^N > n_d^L$, for 162 automatons $n_d^N < n_d^L$, for 1684 automatons $n_d^N = n_d^L$;

- at d = 254 for 154 automatons $n_d^N > n_d^L$, for 126 automatons $n_d^N < n_d^L$, for 1768 automatons $n_d^N = n_d^L$.

Theorem 9. Let $G_d$ , where $d \in \{30, 62, 126, 254\}$, - set of geometrical images of length d of initial discrete deterministic automatons of a kind $A=(\{s_1,s_2,\ldots,s_8\},\{x_1,x_2\},\{y_1, y_2\},\delta, \lambda, s_0 \in \{s_1, s_2, \ldots, s_8\})$ from subclass $H_{11110}$ of (8,2,2)-automatons class and $G_d^1$ $(G_d^0)$ - set of geometrical images of autonomous subautomatons with input signal "1" (signal "0" for $G_d^0$), which are sections of geometrical images from $G_d$ . Then:

- at d = 30 for 988 automatons $n_d^N > n_d^L$, for 90 automatons $n_d^N < n_d^L$, for 970 automatons $n_d^N = n_d^L$;

- at d = 62 for 990 automatons $n_d^N > n_d^L$, for 146 automatons $n_d^N < n_d^L$, for 912 automatons $n_d^N = n_d^L$;

- at d = 126 for 384 automatons $n_d^N > n_d^L$, for 192 automatons $n_d^N < n_d^L$, for 1472 automatons $n_d^N = n_d^L$;

- at d = 254 for 444 automatons $n_d^N > n_d^L$, for 132 automatons $n_d^N < n_d^L$, for 1472 automatons $n_d^N = n_d^L$.

**6. Conclusions.** For each of the 15 non-empty subclasses of the of (8,2,2)-automatons class from the 2 interpolation methods (Newton and Lagrange), the most effective method is determined. Thus, it is shown that the classical methods of interpolation are applicable to the regularization of the laws of functioning of automatons, it is possible to select an effective interpolation method and the model of control and diagnostic objects can be extended to classical interpolation methods.

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