



## TESTING JOINT HYPOTHESES OF THE EFFECTS OF EPICENTERS ON THE INTENSITY OF EARTHQUAKES IN TAIWAN

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### Abstract

The purpose of this paper is to test the joint hypotheses of parameters in the regression equation between intensity and the longitude, latitude, magnitude (Richter magnitude scale,  $M_L$ ), and depth ( $kM$ ) of earthquakes in Taiwan. The sums of squared errors of the unrestricted and restricted models are calculated and F-testis used to check the significance of the testing result. The analyses showed that the parameters of longitude and latitude cannot be neglected in all regions and three eastern coast counties of Taiwan. The adjusted coefficient of determination in the analyses are all higher than 91%, and it means the proposed regression equations can well explain the relationship between intensity with respect to longitude, latitude, magnitude and depth of earthquakes in each area. Three dimensional diagrams are plotted to visualize the intensity for the whole Taiwan and three counties of the eastern coast. From January 1995 to October 2018, there were totally 11,081 earthquakes in Taiwan, and 404 of them caused the intensity equal or above 5.0 (80~250 gal). Although the ratio is only 3.65% (404/11,081), they are categorized as “strong” by the Central Weather Bureau (CWB) of Taiwan, and severe damages or casualties may occur and should be scrutinized.

**Key Words:** Unrestricted model, restricted model, F-test

### Introduction

A null hypothesis with multiple conjectures, expressed with more than one equal sign, is called a joint hypothesis [3]. In this paper, the author uses regression techniques to connect the relationship between the intensity of earthquakes with the locations of their epicenters (longitude, latitude), depths and magnitudes. The intensity is measured by seismometers around the epicenter of an earthquake, and the strongest one is selected to represent the effect of that seismic occasion.

There were 11,081 seismic records in the archive of the Central Weather Bureau (CWB) of the Republic of China (R.O.C., Taiwan) from January 1995 to October 2018. Among them 3,284 were labeled ones, and the remaining 7,797 were categorized as local. Labeled earthquakes are those with magnitude larger than 4.0 and trigger the intensities of two seismometers above 3.0 [1]. There were 404 among these 11,081 earthquakes (3.65%), which caused the intensity more than 5.0 (80~250 gal, 1gal=1cm/sec<sup>2</sup>) and were categorized as “strong.” These strong earthquakes may cause severe damage and casualty, such as the 921 earthquake [6].

The distribution of earthquakes in Taiwan is not uniformly distributed. There are 77.37% of total earthquakes in the three counties of the eastern coast-Yilan, Hualien, and Taitung. In the past 286 months (from January 1995 to October 2018) there are 1,882 seismic records observed in Yilan, 5,119 in Hualien, and 1,612 in Taitung. Hence, in this paper, the intensity studies are focused on the whole of Taiwan as well as the three eastern coast counties.

### Regression analyses

In this section, the relationship it with between intensity and epicenter's longitude, latitude, magnitude (Richter magnitude scale,  $M_L$ )[4,5], and depth ( $kM$ ) will be studied. As long as the equation is found, and substituted with the data of each earthquake, the intensity can be obtained. The earthquake records for the whole 20 municipal cities and counties of Taiwan can be referred to in Table A1. Since there are 11,081 data, only partial of them are shown in Table A1. Since the constant term in a regression equation has no meaning, the author neglects it in all of the analyses.

Two analysis models are studied. The first one is unrestricted model, which includes all factors: longitude, latitude, magnitude ( $M_L$ ), and depth ( $kM$ ). The second one is restricted model, which eliminates two parameters, longitude and latitude, purposely. The *F-test* with values of the sums of squared least squares residuals will be used to test whether the negligence of the parameters of longitude and latitude is significant or not [3].

### The whole data of Taiwan

The general regression of dependent variable (Intensity) with the independent variables (Longitude, Latitude, Magnitude, and Depth) has the following general form:

$$Intensity = \beta_1 Longitude + \beta_2 Latitude + \beta_3 Magnitude + \beta_4 Depth \quad (1)$$

The hypothesis test of  $H_0: \beta_1 = 0$  and  $\beta_2 = 0$  and  $H_1: \beta_1 \neq 0$  or  $\beta_2 \neq 0$

### Unrestricted model for the whole Taiwan

The regression equation of the unrestricted model is:

$$Intensity = -0.037 Longitude + 0.209 Latitude + 0.712 Magnitude - 0.019 Depth \quad (2)$$

$$R_{adj}^2 = 91.88$$

Where  $R_{adj}^2$  is the adjusted coefficient of determination, which measures the proportion of variability in the dependent variable explained by the regression [2,3].

### Restricted model for the whole Taiwan

The regression equation of the restricted model is:

$$Intensity = 0.826 Magnitude - 0.020 Depth \quad (3)$$

$$R_{adj}^2 = 91.52 \%$$

$$F = \frac{(SSE_R - SSE_U) / J}{SSE_U / (N - K)} \quad (4)$$

Where  $SSE_R$  is the sums of squared least squares residuals from the unrestricted model.  $SSE_U$  is the sums of squared least squares residuals from the restricted model.  $J$  is the number of restrictions,  $J=2$  in this analysis.  $K$  is the number of coefficients in the unrestricted model,  $K=4$ .  $N$  is the number of observations.

$$F = \frac{(7623.0677 - 7315.7748)/2}{(7315.7748/(11081 - 4))} = 232.64 > F_c = 3.0$$

$F > F_c$ , reject  $H_0: \beta_1 = 0$  and  $\beta_2 = 0$ , and it means that at least one of the coefficients of longitude and latitude is not zero. More precisely, during the regression, the parameters of longitude or latitude for the whole Taiwan cannot be eliminated simultaneously.

### Intensity map for the whole Taiwan

There were totally 11,081 earthquakes from January 1995 to October 2018. Their longitude, latitude and intensity are plotted as follows:

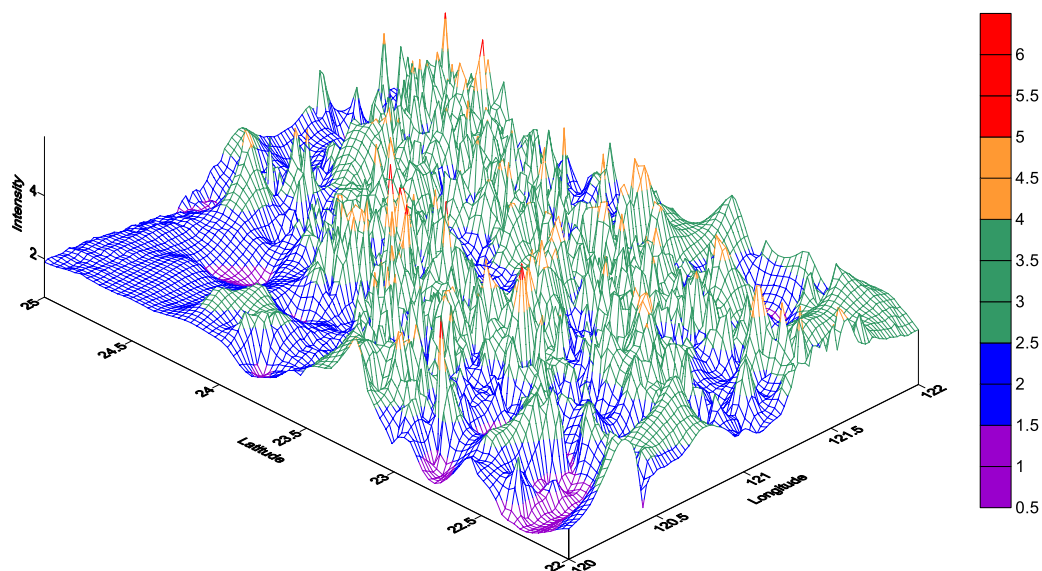


Figure 1: Intensity map for the whole Taiwan

From the above figure, one finds most of the intensities of earthquakes in Taiwan are below 4.0. But the 921 earthquake ( $M_L=7.3$ ), which occurred on September 21, 1999, and its aftershocks induced some intensities as high as 7.0 (more than 400 gal) [1, 6].

### Yilan

#### Unrestricted model for Yilan

The regression equation of the unrestricted model for Yilan is:

$$Intensity = 0.195Longitude - 0.905Latitude + 0.507Magnitude - 0.019Depth \quad (5)$$

$$R_{adj}^2 = 93.18 \%$$

### Restricted model for Yilan

The regression equation of the restricted model for Yilan is:

$$Intensity = 0.938Magnitude - 0.025Depth \quad (6)$$

$$R_{adj}^2 = 91.98 \%$$

$$F = ((1368.87095 - 1162.24953)/2) / (1162.24953 / (1882 - 4)) = 166.9328 > F_c = 3.0$$

$F > F_c$ , reject  $H_0: \beta_1 = 0$  and  $\beta_2 = 0$ , and it means that at least one of the coefficients of longitude and latitude is not zero. More precisely, during the regression, the parameters of longitude or latitude for Yilan cannot be eliminated simultaneously.

### Intensity map for Yilan

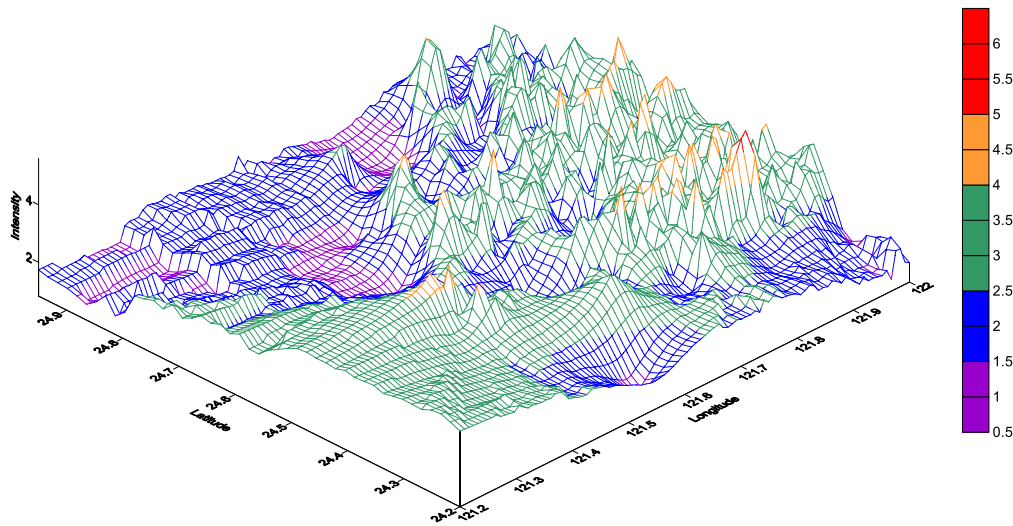


Figure 2: Intensity map for Yilan

From the above figure, one finds most of the intensities of earthquakes in Yilan are below 5.

### Hualien

#### Unrestricted model for Hualien

The regression equation of the unrestricted model for Hualien is:

$$Intensity = -0.096Longitude + 0.506Latitude + 0.745Magnitude - 0.032Depth \quad (7)$$

$$R_{adj}^2 = 93.18 \%$$

#### Restricted model for Hualien

The regression equation of the restricted model for Hualien is:

$$Intensity = 0.904Magnitude - 0.036Depth \quad (8)$$

$$R_{adj}^2 = 92.46 \%$$

$$F = ((3065.60867 - 2922.4555)/2) / (2922.4555 / (5119 - 4)) = 125.27 > F_c = 3.0$$

$F > F_c$ , reject  $H_0: \beta_1 = 0$  and  $\beta_2 = 0$ , and it means that at least one of the coefficients of longitude and latitude is not zero. More precisely, during the regression, the parameters of longitude or latitude for Hualien cannot be eliminated simultaneously.

### Intensity map for Hualien

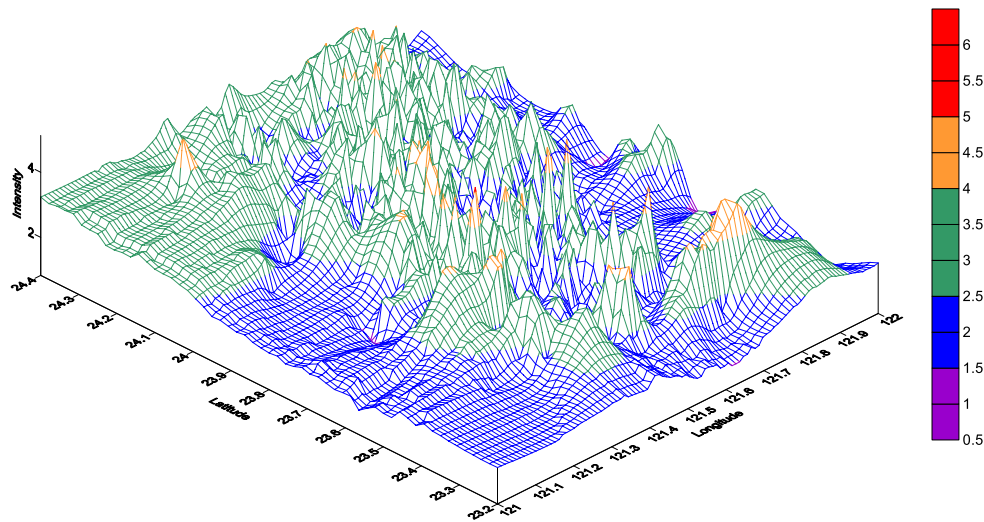


Figure 3: Intensity map for Hualien

From the above figure, one finds most of the intensities of earthquakes in Hualien are below 5.

### Taitung

#### Unrestricted model for Taitung

The regression equation of the unrestricted model for Taitung is:

$$Intensity = -0.017Longitude + 0.098Latitude + 0.730Magnitude - 0.017Depth \quad (9)$$

$$R_{adj}^2 = 92.72 \%$$

#### Restricted model for Taitung

The regression equation of the restricted model for Taitung is:

$$Intensity = 0.777Magnitude - 0.018Depth \quad (10)$$

$$R_{adj}^2 = 92.69 \%$$

$$F = ((985.78565 - 980.4423)/2)/(980.4423/(1612 - 4)) = 4.38 > F_c = 3.0$$

$F > F_c$ , reject  $H_0: \beta_1 = 0$  and  $\beta_2 = 0$ , and it means that at least one of the coefficients of longitude and latitude is not zero. More precisely, during the regression, the parameters of longitude or latitude for Taitung cannot be eliminated simultaneously.

### Intensity map for Taitung

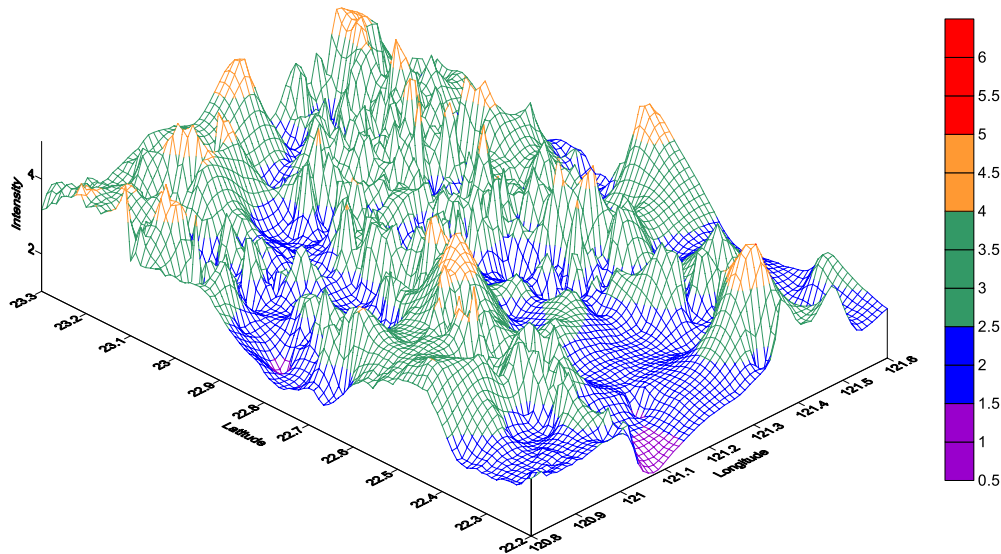


Figure 4: Intensity map for Taitung

From the above figure, one finds most of the intensities of earthquakes in Taitung are below 5. The results of the above analyses are summarized in Table 1 as follows:

Table 1: The regression equations of intensity related to longitude, latitude, magnitude, and depth of earthquakes for the whole Taiwan and three counties in the eastern coast

Area	Regression	$R_{adj}^2$ (%)
Whole Taiwan	$Intensity = -0.037Longitude + 0.209Latitude + 0.712Magnitude - 0.019Depth$	91.88
Yilan	$Intensity = 0.195Longitude - 0.905Latitude + 0.507Magnitude - 0.019Depth$	91.98
Hualien	$Intensity = -0.096Longitude + 0.506Latitude + 0.745Magnitude - 0.032Depth$	93.18
Taitung	$Intensity = -0.017Longitude + 0.098Latitude + 0.730Magnitude - 0.017Depth$	92.72

## Conclusions:

Based on the joint hypotheses testing in the section above, the following results can be obtained: The parameters of longitude or latitude cannot be eliminated simultaneously for the whole Taiwan and all three counties in the eastern coast although the values seem to be small.

The joint hypotheses testing of the whole region of Taiwan and its three counties in the eastern coast rejects  $H_0: \beta_1 = 0 \text{ and } \beta_2 = 0$  and  $H_1: \beta_1 \neq 0 \text{ or } \beta_2 \neq 0$  cannot be rejected.

The adjusted coefficient of determination  $R_{adj}^2$  in all of the regression equations are higher than 91% in Table 1. It means the intensity can be strongly explained by the regression equations.

The intensities of earthquakes are plotted by three dimensional diagrams for the whole Taiwan and three eastern coast counties. Most of the intensities of earthquakes are below 5.0 according to the records in the Central Weather Bureau (CWB) and are observed in the diagrams.

There were 404 earthquakes is in the range of 80~250 gal (1 gal= 1cm/sec<sup>2</sup>) [1] among 11,081 records (404/11,081=3.65%), which induced intensity equal or above than 5.0, and were categorized as “strong” by the CWB.

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## Appendix 1: Earthquakes records from January 1995 to October 2018 (286 months)

Table A1: Properties of earthquakes in Taiwan from Jan. 1995 to Oct. 2018 (only partially shown)

Number	Longitude	Latitude	$M_L$	Depth (kM)	Intensity
1	121.7	24.96	4.5	91.5	2
2	121.43	23.68	5.1	3.8	6
3	120.76	23.3	4.5	14.3	3
4	120.19	23.05	4.2	16	4

5	121.92	23.76	5.2	24.6	3
6	121.74	24.03	4.7	27	3
7	121.69	24.2	5.8	21.7	5
8	121.7	24.26	4.4	25.7	3
9	120.23	23.06	4	12.1	4
10	121.46	23.06	4.9	16.5	4
11	121.52	23.78	4.8	17.9	3
12	121.86	24.64	5.6	76	4
13	122.43	23.94	5.9	14.6	3
14	121.02	21.83	5	29.8	3
15	121.45	23.76	4.3	17.7	5
16	120.46	23.25	4.3	6.5	4
.	.	.	.	.	.
.	.	.	.	.	.
11065	120.47	22.12	4.4	37.3	2
11066	121.35	23.11	3.5	18.7	2
11067	121.01	24.72	3.6	6.1	4
11068	121.27	22.84	3.5	17.5	2
11069	121.37	22.91	4.4	20.8	4
11070	122.08	24.89	4.4	13.7	3
11071	120.71	23.1	3.6	6.3	2
11072	121.39	24.69	3.1	27.2	1
11073	121.62	24.05	3.1	7.1	2
11074	121.4	24.09	4.4	63.4	2



11075	122.64	24	6	31.3	3
11076	122.66	24	5.7	29.4	3
11077	120.38	23.21	2.9	9.9	2
11078	122.01	23.5	4	20.3	2
11079	121.71	24	3.1	21	2
11080	121.01	23.98	3.3	16.1	2
11081	121.4	23.75	4.2	21.5	4