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REBIRTH OF MATHEMATICS IN THE 19TH CENTURY

Boniello Carmine
University of Salerno
Fisciano (Italy)

Abstract

XIX century mathematics became something visibly different from what it had been in the XVIII century, in the post-Newtonian heroic era. It was born and developed within it a new, rigorous and severe spirit. The discoveries and methods were perfected more and more. The fields of mathematical research multiplied, preparing the conceptual tools for subsequent discoveries in the physical field and great applications in engineering. The work was structured in such a way as to highlight not only the importance of XIX mathematics and its countless discoveries but to highlight that these discoveries have marked the mathematical discoveries of the following centuries.

Keywords: Mathematics, history of mathematics, innovation of mathematics

1. The evolution of mathematics in the XIX century

The development of mathematics in the XIX century occurred with increasing rapidity, similar to that of the rest of science. The nineteenth century is all pervaded by a need for rigor that will lead mathematicians to grapple with the conceptual knots that have not yet been resolved or to tackle long-unsolved problems, before devoting themselves to discovering the "new". Thus, the re-foundation of the analysis on the formal concept of limit will dissolve the mists that still surrounded the concepts of infinity, infinitesimal, convergence and divergence of a series, continuity of a function, and so on. Similarly, the theory of algebraic equations will provide a theoretical explanation of the centuries-old series of failures recorded in the search for general solution formulas for equations of higher than fourth degree. Geometry, then, will close the millenary efforts to demonstrate the Euclidean postulate of the parallel, through the construction of non-Euclidean geometries and the justification of their logical legitimacy, bringing out, precisely as a consequence of this logical-critical effort, even a new face of this discipline.

The nineteenth century is also the century in which mathematical logic was born, in an explicit and systematic way, a prelude to that deepening in the "research on the foundations" that will characterize the last decades of the century, with the attempt of a purely logical foundation of arithmetic operated by Frege[1] and with Cantor's[2] creation of abstract set theory. At the beginning

[1] Friedrich Ludwig Gottlob Frege (1848 - 1925) was a mathematician, logician and philosopher German, father of modern mathematical logic, and then controversially also proclaimed father of analytic philosophy from the exegetical work of Michael Dummett, as well as a scholar of epistemology, the philosophy of mathematics and the philosophy of language. Frege is considered almost unanimously by today's critics as one of the greatest logicians after Aristotle, and is the father of formal thought of the XX century.

[2] Georg Ferdinand Ludwig Philipp Cantor (1845 - 1918) was a German mathematician, the father of modern set theory. Cantor has extended set theory to include within it the concepts of transfinite numbers, cardinal numbers and ordinal. Cantor recognized that infinite sets can have different cardinality, separated the sets into countable and more than countable and proved that the set of all rational numbers Q is s countable while the set of all real numbers R it is more than countable, thus demonstrating that there are at least two orders of infinity. He also invented the symbol that is used today to indicate real numbers. The method he used to conduct his proofs is known as the Cantor diagonal

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of the XIX century, Gauss[3] was the only one to have an intuition of what was about to happen, but his Newtonian reserve prevented him from communicating his predictions to Lagrange[4], Laplace[5] and Legendre[6]Although these great mathematicians lived in the first thirty years of the nineteenth century, most of their works appear to us today as preparatory works; Lagrange, with his theory of equations, paved the way for Abel[7] and Galois[8]. Laplace, with his works on the differential equations of Newton's astronomy (including the theory of gravitation) did nothing but prelude to the phenomenal development of mathematical physics in the nineteenth century, and finally Legendre's research on integral calculus, pointed to Abel and to Jacobi[9] one of the most fertile fields of investigation that analysis has ever known. In this tumultuous evolution it is possible, however, to identify the growth and evolution of some lines of research and thought, and the birth of new branches on the old trunk of a secular science; all phenomena that have brought mathematics to the structure it has in the present century, or rather to the structure it had towards the beginning of the development of electronic means of calculation. These, with their appearance and their diffusion, have revolutionized the panorama of a science that appears at first sight as rigid and almost crystallized, and which instead often presents surprising turning points. To get a first idea of the change and progress that mathematics has made in the XIX century, just follow the evolution of geometry with the flowering of projective and descriptive

method. Later, he tried in vain to prove the continuum hypothesis. Cantor formulated a very important principle for the definition of real numbers, called the localization principle, which is also fundamental to be able to operate on the aforementioned numerical field.

[3] Johann Friedrich Carl Gauss (1777-1855) was a mathematician, astronomer and physicist German, who gave contributions determinants mathematical analysis, number theory, statistics, numerical calculation, differential geometry, geodesy, geophysic, magnetism, electrostatics, astronomy and optics.

[4] Joseph-Louis Lagrange, (1736-1813), was an Italian mathematician and astronomer active, in his scientific maturity, for twenty-one years in Berlin and twenty-six in Paris. He is unanimously considered one of the greatest and most influential European mathematicians of the eighteenth century. His most important work is the Mécanique analytique, published in 1788, with which rational mechanics was conventionally born. In mathematics, he is remembered for his contributions to number theory, for being one of the founders of the calculus of variations, deducing it in the "Mecánique", for having outlined the foundations of rational mechanics, in the formulation known as Lagrangian mechanics, for the results in the field of differential equations and for being one of the pioneers of group theory. In the field of celestial mechanics he conducted research on the phenomenon of lunar libration and, later, on the movements of the satellites of the planet Jupiter; he investigated the problem of the three bodies and their dynamic equilibrium with the rigor of mathematical calculation. His pupils were Jean-Baptiste Joseph Fourier, Giovanni Plana and Siméon-Denis Poisson.

[5] Pierre-Simon Laplace, Marquis de Laplace (1749 - 1827), was a mathematician, physicist, astronomer and noble French. He was one of the leading scientists of the Napoleonic period, appointed Minister of the Interior in 1799 by Napoleon, who in 1806 conferred on him the title of Count of the Empire, then also appointed Marquis in 1817, after the restoration of the Bourbons. He made fundamental contributions in various fields of mathematics, astronomy and probability theory and was one of the most influential scientists of his time, also for his contribution to the affirmation of determinism. He put the final twist on mathematical astronomy, summarizing and extending the work of his predecessors in his five-volume work Mécanique Céleste (Celestial Mechanics) (1799-1825). This masterpiece transformed the geometric study of mechanics developed by Newton into one based on mathematical analysis.

[6] Adrien-Marie Legendre (1752-1833) was a French mathematician. A disciple of Euler and Lagrange, he published a now classic work on geometry, Élements de géométrie. He also made significant contributions in differential equations, the calculus, the theory of functions and number theory with Essai sur la théorie des nombres (1797-1798), in 1782 he was granted the prize offered by the Berlin Academy for his bullet studies. He expanded his three-volume treatise Exercises du calcul intégral (1811 - 1819) in the new Traité des fonctions elliptiques et des intégrales eulériennes (1825 - 1832), again in three volumes. He then reduced the elliptic integrals to three standard forms, but their direct inversion, due to Abel and Jacobi , rendered his work useless. He also invented the Legendre polynomials in 1784 while studying the attraction of spheroids. In number theory, he also proved Fermat 's last theorem in the particular case n = 5 and proved the irrationality of π^2.

[7] Niels Henrik Abel (1802 -1829) was a Norwegian mathematician, best known for his fundamental contributions to algebra and the theory of functions; also remembered for the award that bears his name.

[8] Évariste Galois (1811 - 1832) was a French mathematician . While still an adolescent, he was able to determine a necessary and sufficient condition for a polynomial to be resolvable by radicals, thus solving a problem over 350 years old. His work led to the birth of Galois theory and group theory, two important branches of abstract algebra, and the subfield of Galois connections. He died at the age of 20, from injuries sustained in a duel.

[9] Carl Gustav Jacob Jacobi (1804 - 1851) was a German mathematician and teacher.

geometry. In other words, the development of geometry in the XIX century constitutes a paradigm of the evolution of mathematics in that period. The central body of XIX century mathematics, however, is constituted by mathematical analysis and the progress of this branch of science contributed in a fundamental and irreplaceable way to characterize the mathematical method of subsequent periods and the physiognomy of mathematics in our time. The numerous disciplines that developed in this century became autonomous, each with its own terminology and methodology. Personal researches resulted in the formulation of ever more specialized and difficult problems, which required ever more ingenious ideas, ever richer inspirations, and ever less perspicuous demonstrations; to make progress, mathematicians had to acquire a broad theoretical background and great technical skill. Among the branches of mathematics that are developing in this century we want to focus on algebra, on the calculation of probabilities, and on formal logic. And we only remember these because their appearance seems remarkably symptomatic to us, and their character helps to better understand the development of later mathematics, particularly that of the XX century.

2. The theory of algebraic equations

There remained unanswered, at the beginning of the XIX century, some long-standing problems, dating back to the XVI century and among these those concerning the solution of algebraic equations. Gauss had rigorously proved that there exists at least one complex solution of an algebraic equation whose coefficients are complex numbers; however, the question concerning the algebraic tools that allow to express the solutions of the equations as a function of their coefficients remained unsolved. Probably the possibility of expressing the solutions of the equations of the first four degrees by means of quadratic and cubic radicals (that is, up to and including the fourth degree) had led mathematicians to insist on the search for the solutions of the general algebraic equations by means of this type. A first negative result was achieved by Paolo Ruffini[10], and later perfected by Abel; both proved, that it is not possible to express the roots of a general algebraic equation of degree higher than the fourth through expressions that contain functions of the coefficients of a given type precisely, rational functions or obtained with roots of any integer order infinite number. The investigation of these problems led to the analysis of the structure of certain operations and their composition; in fact, the idea and the custom of working on certain "things" that are not numbers, for which they could nevertheless be defined as "products", made their way; moreover, for these new "entities" the product operation does not have all the properties of the product between two numbers, for example it is not, in general, commutative. The concept of "group" was therefore constructed; at the beginning, this concept was linked to the concept of a set of operations in a finite number, such as, for example, the operations of substitution between a finite number of elements. But the concept then extended to the consideration of operations in infinite numbers. Galois brilliantly linked the concept of group with the problem of the solution of algebraic equations, and, in particular, applied it to the

[10] Paolo Ruffini was born in Valentano to Ruffini, a doctor who had moved there from Reggio Emilia, and to Maria Francesca Ippoliti from Poggio Mirteto. In 1783 he enrolled at the University of Modena, where he mainly studied mathematics and medicine, but also literature and philosophy. He was a pupil of Luigi Fantini, a well-known expert in geometry, and of Paolo Cassiani, professor of analysis.

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solution of these equations by means of certain expressions and certain functions (radicals, for example), having properties and well-defined formal laws. In this order of ideas, the work of Galois is fundamental and definitively closes a centuries-old series of research, opening in turn a new field of study. The concept of the group evolved more and more rapidly and became increasingly detached from the particular realization that had given occasion to its introduction and its use.

3. The concept of limit

A first important moment in the development of mathematical analysis in the XIX century is undoubtedly constituted by the clarification of the basic concepts of that doctrine which was called "infinitesimal calculus". These concepts were originally introduced on the basis of mechanical or geometric intuition and were justified in part on the same basis and in part also on reasonings of which further analysis often highlighted the lack of rigor, even when they reached to correct and acceptable results. One of the fundamental concepts of classical mathematical analysis is undoubtedly that of limit; it is the basis of the very concept of real number, and of the concept of convergence of a sequence and of a series, and of the procedure that leads to the integration of a function in an interval. To fix the ideas, we can focus on the concept of limit of a succession of numbers; in this case, an approximate description of the concept itself is made by saying that a number is the limit of a sequence when its elements "approach indefinitely" to this limit. To this concept of "indefinite approach" obviously corresponds that of "indefinite distancing", when it is a question of those limits that it is convenient to indicate as infinite. For the clarification of the meaning of that adverb "indefinitely", which enters the classical definitions of convergence to a limit, difficulties of a logical nature arise. In fact, the imagination represents an indefinite repetition of acts or thoughts that lead to a distancing up to unreachable distances, or to a shrinking below any possibility of observation. But obviously these images are not sufficient to ground the concept clearly and to allow a rigorous deduction. The answer to the requests for clarity and rigor was mainly given thanks to the analyzes of Cauchy[11] and Weierstrass[12]; we therefore arrived at the classical attitude in which a number L is defined as the limit of a given succession of numbers if, fixed in any way a positive number ε, there exists an integer n such that the element of the succession having index nor do all its successors have a distance from L less than ε. Similarly, we can arrive at the definition of the limit of an infinite

[11] Augustin-Louis Cauchy (1789 - 1857) was a mathematician and French engineer. He started the project of the rigorous formulation and demonstration of the theorems of the infinitesimal analysis based on the use of the notions of limit and continuity. He also gave important contributions to the theory of functions of complex variable and the theory of differential equations. The systematicity and the level of these works places it among the fathers of the mathematical analysis. Cauchy's genius is evident in his simple solution to Apollonius 's problem, namely the description of a circle touching three other data circles he discovered in 1805, the generalization of Euler 's characteristic for polyhedra in 1811, and many other problems solved elegantly. Of great importance are his writings on wave propagation, thanks to which he obtained the Institute's Grand Prix in 1816. His greatest contributions to mathematics are enshrined in the rigorous methods he introduced. This is found mainly in his three major treatises: Cours d'analyse de l ' École Polytechnique (1821); Le Calcul infinitésimal (1823); Leçons sur les applications de calcul infinitésimal; La géométrie (1826-1828); and also in his Courses of mechanics (for the École Polytechnique), Higher algebra (for the Faculté des Sciences), and of Mathematical physics (for the Collège de France).

[12] Karl Theodor Wilhelm Weierstrass (1815 -1897) was a German mathematician often cited as the "father of modern analysis". Despite leaving university without a degree, he studied mathematics and trained as a school teacher, eventually teaching mathematics, physics, botany and gymnastics. He later received an honorary doctorate and became professor of mathematics in Berlin. Among many other contributions, Weierstrass formalized the definition of the continuity of a function, proved the intermediate value theorem and the Bolzano–Weierstrass theorem, and used the latter to study the properties of continuous functions on closed bounded intervals.

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algorithm different from the sequence: for example, series, infinite product, continued fraction, etc. In a similar way we arrive at the definition of the limit of a succession, when it is infinite; and, even with considerations that follow the same spirit, even if slightly different, we can define the limit of a real function of a real variable at a point, and on the basis of this, we can define the concept of continuity of such a function. The same line of development has also followed the definition of the concept of "integral" of a real function of the real variable. In this order of ideas, Cauchy's research is in the same direction as those already begun by the tuscan mathematician Mengoli[13]. Riemann[14] further specified the conditions under which it is possible to speak of the existence of the integral of a real function of a real variable. This concept obviously generalizes the concepts that had already guided Archimedes[15] and then Cavalieri[16] in the search for the area of a plane figure whose boundary is a line not completely formed by straight segments. The clarification of the concept of limit of a sequence also brought with it the rigorous construction of the concept of real number; that is, it led to the rigorous solution of the problem of representing the geometric continuum with suitable analytical tools.

4. The evolution of the concept of function

Parallel to the clarification of the concept of limit, there has been a deepening of the concept of function, which is one of the fundamental concepts of mathematical analysis. As is known, it is closely connected with that of correspondence. In fact, given two sets A and B, when there is a correspondence that makes one and only one element of B correspond to an element of A, but not necessarily to all the elements of the set itself, it is said that this second is a function of the first. In the classical elementary conception the two sets A and B were coincident with the set of real numbers or were certain subsets of it; a number belonging to the set A was called "independent variable" and the correspondent of the set B "dependent variable"; the concept of numeric function was closely associated with the concrete operations that lead from an element x

[13] Pietro Mengoli (1626 - 1686) was an Italian mathematician . He studied with Bonaventura Cavalieri and took over the teaching of mathematics at the University of Bologna. His studies are halfway between Cavalieri's method of indivisibles and those of Leibniz and Newton. Among other things, he wrote the Geometriae speciosae elementa (1659), anticipating Cauchy in relation to the concept of limit and definite integral.

[14] Georg Friedrich Bernhard Riemann (1826 - 1866) was a German mathematician and physicist. He contributed significantly to the development of the mathematical sciences. In 1859 he published a ten-page essay on the notes of the Prussian Academy of Sciences, entitled Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse, the only one that Riemann wrote on the theory of numbers. Among other things, what is now known as the Riemann Hypothesis was buried in it. Riemann was very religious. His contribution to mathematics was important, but we should also remember the other studies he carried out from the early stages of his career, which dealt with physical problems, such as magnetic fluids, Faraday's law of induction, as well as themes of natural philosophy, metaphysics, theory of knowledge and psychology.

[15] Archimedes of Syracuse (287 BC-212 BC) was a Sicilian mathematician, physicist and inventor. Considered as one of the greatest scientists and mathematicians in history, he contributed to advance knowledge in areas ranging from geometry to hydrostatics, from optics to mechanics: he was able to calculate the surface and volume of the sphere and intuited the laws that they regulate the buoyancy of the bodies; in the engineering field, he discovered and exploited the operating principles of levers and his very name is associated with numerous machines and devices, such as the Archimedes screw, demonstrating its inventive ability; still surrounded by an aura of mystery are the war machines that Archimedes would have prepared to defend Syracuse from the Roman siege. His life is remembered through numerous anecdotes, sometimes of uncertain origin, which contributed to building the figure of the scientist in the collective imagination. For example, the exclamation èureka ! Has remained famous over the centuries! attributed to him after the discovery of the principle on the buoyancy of bodies that still bears his name

[16] Francesco Francesco Bonaventura Cavalieri (1598-1647) was a mathematician and academic Italian. Cavalieri's fame is mainly due to the indivisibles method, used to determine areas and volumes: this method represented a fundamental step for the future elaboration of the infinitesimal calculus. It was above all Galilei himself who pushed Cavalieri to deal with the problems of infinitesimal calculus. Subsequently, it constituted a point of reference for some of the geometric research of the young Evangelista Torricelli.

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of the set A to the number that corresponds to it. These operations were obviously those taken into consideration by the mathematics of time: rational operations (sum, product, difference, division) and solutions of algebraic equations. This rather narrow vision then expanded in various directions, which I will briefly summarize. A first step in the direction of broadening the concept of function was taken by admitting more general algorithms for calculating the value corresponding to a certain real number, or to a certain set of real numbers. In this order of ideas it is worth remembering the step taken by Lagrange, with the introduction of his concept of "analytic function".

With this attitude, the meaning and role of the classic formulas, known for some time, which are usually called Taylor[17] or Mac Laurin[18] formulas, is somewhat changed. Indeed, such formulas had been considered as means of calculating the values of a function that was supposed to be known and given; means that could also involve infinite algorithms such as series. The novelty of Lagrange's idea consists in assuming these formulas as definitions of the functions in question, and of other functions whose values are thus defined and calculated methodically with much more general algorithms than those used up to that time. A further step in this direction was taken by Weierstrass, who methodically used a determined algorithm (the power series) to define the complex function of the complex variable, which Cauchy had introduced. Further steps were taken by assuming more general infinite algorithms, such as the trigonometric series studied by Joseph Fourier[19]. A second step in the direction of the generalization of the concept of function was accomplished by taking into consideration functions no longer given directly by algorithms, but by mathematical conditions. Among the conditions most frequently contemplated we recall here the differential equations, that is the links between a function and its derivatives; in particular, those obtained with the integration operation can also be attributed to the class of these functions. Also in this order of ideas we must remember the implicit functions, that is those defined by one or more links between the variables, and again the algebraic functions, which are obtained when the link or links are constituted by algebraic equations. Finally, let us recall the conditions that can be posed by imposing on a function to render a certain definite integral maximum (or minimum). Problems of this kind were at the origin of the calculus of variations; their solution had been traced by Euler to the solution of certain differential equations, and the concepts related to the calculus of variations had a very important place in the mathematical

[17] Frederick Winslow Taylor (1856 -1915) was an American mechanical engineer. He was widely known for his methods to improve industrial efficiency. He was one of the first management consultants. Taylor was one of the intellectual leaders of the Efficiency Movement and his ideas, broadly conceived, were highly influential in the Progressive Era (1890s–1920s). In 1911, Taylor summed up his efficiency techniques in his book *The Principles of Scientific Management* which, in 2001, Fellows of the Academy of Management voted the most influential management book of the twentieth century. His pioneering work in applying engineering principles to the work done on the factory floor was instrumental in the creation and development of the branch of engineering that is now known as industrial engineering. Taylor made his name, and was most proud of his work, in scientific management; however, he made his fortune patenting steel-process improvements.

[18] Colin Maclaurin (Scottish Gaelic : Cailean MacLabhruinn; Kilmodan, February 1, 1698-Edinburgh, June 14, 1746) was a Scottish mathematician. Certainly one of the most brilliant mathematicians of the time, he made a notable contribution to mathematical analysis and contributed above all to the development of the "series of functions". Its name is linked to a particular case of the Taylor series called "Maclaurin series ". His studies led him to elaborate the Euler-Maclaurin formula, and he also dealt with the calculation of the determinants, the orbit of the sun, the structure of the hives, the effect of work on bodies and the phenomenon of the tides.

[19] Jean Baptiste Joseph Fourier (1768-1830) was a French mathematician and physicist, best known for his famous series and transform and for his law on the conduction of heat. His major contributions include: the theorization of the Fourier series and the consequent Fourier transform in mathematics, the formulation of the linear constitutive law for thermal conduction and Fourier's law in thermodynamics.

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physics of the nineteenth century, which used these concepts for the formulation of general laws. It seems natural, therefore, that the final step was to conceive any correspondence, not linked, at least at first sight, to certain calculation algorithms, to certain conditions expressible with equations or, in general, with mathematical relationships.

5. The construction of the complex field and the functions of a complex variable

The broadening of the views of algebra, geometry and mathematical analysis brought to the fore the problem of the rigorous foundation of the concept of complex number. This problem can be traced back to the sixteenth century, because Bombelli[20] had already recognized the opportunity to broaden the field of real numbers, introducing entities that satisfy calculation laws different from those that hold true for real numbers. In this way entities were constructed, which can be represented for example in the form of the sum of two addends: a+ib, where a and b are real numbers and i is a new entity, called "imaginary unit", which satisfies the law of calculation i*i=-1; law contrary to those that hold in the real field, in which the square of any number is always positive or null. It was found that, by using these new entities, called "complex numbers", it was possible to satisfy those algebraic equations which in the real field have no solutions. But the legitimacy of use of these entities, and the logical coherence of the rules that govern the operations on them, remained suspended until it was possible, thanks to Gauss, Argand[21] and Augustin Cauchy, to give a representation of them through the points of a plane, and to give models of the operations defined on them by means of geometric contents. We also owe to Gauss the first rigorous proof of the theorem which is usually called the "fundamental theorem of algebra", and which affirms the existence of a solution, in the field of complex numbers, of every algebraic equation whose coefficients are also real or complex. The rigorous justification of the laws of calculus of complex numbers soon found an ingenious extension in the concept of complex function of the complex variable. In this field the work of Cauchy is fundamental, who had the idea of constructing functions that present themselves as a natural and immediate generalization to the complex field of real functions of the real variable, and in particular present at every point and for every value only one derivative of the variable. Cauchy called these functions "monogene", and characterized them through certain differential conditions that are usually called "Cauchy conditions" or even "Cauchy-Riemann conditions" from the name of the great German mathematician who also made fundamental contributions to their theory. To

[20] Bombelli Raffaele (Bologna 1526 - Rome 1572), Italian mathematician, one of the great algebraists of the Renaissance. He was responsible for introducing imaginary numbers and perfecting the theory of third and fourth degree equations treated by Gerolamo Cardano, Niccolò Tartaglia and Ludovico Ferrari. Bombelli privately completed architectural and engineering studies and worked as an engineer, mainly working on the Val di Chiana reclamation project. His contribution to mathematics is contained in a treatise on algebra in 3 volumes (the original project included 5), published the year of his death and entitled Algebra. In addition to the detailed description of operations between imaginary numbers, the treatise includes computation rules for negative numbers and related proofs. The value of the work was also recognized by the great German mathematician and philosopher Gottfried Leibniz, who defined the author as a master of analytical art.

[21] Jean-Robert Argand (1768 - 1822) was a non-professional swiss mathematician. In 1806, while he was the manager of a bookshop in Paris, he published a book at his own expense in which the idea of the geometric interpretation of complex numbers was expounded. Thanks to this study, he and Carl Friedrich Gauss have been given the title of the Argand-Gauss plane (Complex plane), a graph that represents the real part of the complex number on the abscissa and the imaginary part on the ordinate, thus transforming a complex number into a vector. Among his

Cauchy we owe the fundamental theorems of the theory of these functions; and it should be remembered that even today these constitute a fundamental tool for the study of many problems of mechanics and mathematical physics. The theory of complex variable functions has also made it possible to answer various problems, which had their historical origins in Greek mathematics.

6. Boolean algebra and formal logic

In the XIX century formal logic, which constitutes one of the most important branches of modern mathematics, underwent a particular development. The origins of this doctrine can be searched very far, strictly speaking even the classical formal logic had the characteristics of a kind of calculus of prepositions, because it taught to deduce true prepositions from others supposed or accepted as true, starting from the only form of the prepositions themselves. However, although classical formal logic (called "minor") basically aimed at the manipulation of propositions starting from their form alone and not from their contents, it lacked the presence of abstract artificial symbolism, which is instead characteristic of modern logic. understood, and which approaches this doctrine with abstract algebra. Instead, we find very clear premonitions of the modern situation in Leibniz[22] who predicted the time in which two philosophers would no longer dispute with very long and inconclusive speeches but seated at a table they would pick up the pen saying "Calculemus". It could be said that Leibniz's attitude manifests quite well those requirements of univocity of symbols, of mechanicality and certainty of deductions, which are among the typical characteristics of mathematical language. These considerations lead to at least partially explain the adoption of an ideographic symbolism informal logic; symbolism that was able to overcome the misunderstandings of living language, and allowed to enunciate the laws of deduction as laws of an algebraic calculation. George Boole[23], is usually credited with initiating this attitude. He adopts symbols to express ideas directly, and defines operations, to which he still retains the name of "sum" and "product", of which he detects the "strange" properties with respect to the homonymous operations that hold for numbers. He thus created a real algebra, which is the germ of the doctrine still today called "Boolean algebra", and which finds many different theoretical and practical applications. Therefore, the birth of this "strange" algebra is accompanied by the birth of the other algebras we have already mentioned; In this way, the evolution of thought that was to lead mathematicians to focus their attention more on the structures of operations than on the objects on which they operate is increasingly manifested. The creation of new tools for logic was stimulated by the studies that led first to make the traditional arguments of infinitesimal calculus rigorous, and then to probe the foundations of geometry and arithmetic.

contributions it is also necessary to remember a (not completely correct) proof of the Fundamental Theorem of Algebra; Argand seems to have been the first to deal with the case where coefficients can also be complex numbers.

[22] Gottfried Wilhelm von Leibniz (1646- 1716) was a philosopher , mathematician , scientist , logician , theologian , linguist , glottoteta , diplomat, jurist , historian , German magistrate . Considered the precursor of computer science, neuroinformatics and automatic computing, he was the inventor of a mechanical calculator called the Leibniz machine; moreover, some areas of his philosophy opened numerous glimpses on the dimension of the unconscious that only in the twentieth century, with Sigmund Freud, will we try to explore.

[23] George Boole (1815 - 1864) was a British mathematician and logician, considered the founder of mathematical logic. Father of mathematics Alicia Boole, his work also influenced sectors of philosophy and gave birth to the school of algebrists of logic.

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7. Cantor and set theory

Fundamental in the study of sets was the work of Cantor Georg[24]who left an indelible imprint in mathematics by creating what is called "set theory". In a first approach, elementary and intuitive, the concept of the whole can be taken as primitive; therefore it is not possible to give a formal definition, but it is necessary to leave the understanding of its meaning to the knowledge that comes to us from common language, and limit ourselves to enunciating synonyms (such as collection, collection, class, family, etc.) and to verify that the term is used in certain ways. Even elementary mathematics offers us quite simple examples of infinite sets: among others, the set of integers and the set of points of the line or of a segment. It is worth remembering that the problem concerning sets with infinite elements had already been encountered in the centuries preceding the nineteenth; we recall among other things the famous passage by Galileo in which it is observed that the concepts of whole and part, valid and apparently clear when dealing with material entities and finite sets, must be analyzed with rigorous logical precautions when dealing with sets infinite. One of the fundamental concepts on which the Cantorian theory is based is that of the "power" of a whole. Approximately, it could be said that it constitutes the immediate generalization of the concept of integer: supposing we know what we mean by speaking of a finite set (i.e. with a finite number of elements), supposing we know what we mean by one-to-one correspondence between two sets, the fact that a one-to-one correspondence can be established between them can be considered as a different way of saying that they have the same cardinal number of elements. With an ingenious generalization of these concepts, Cantor came to construct a theory of cardinal numbers that he called "transfinites", and to establish an arithmetic of these, that is, a set of calculation rules. He also pushed forward the analysis of the geometric continuum from this point of view, with results at first sight paradoxical, but profoundly innovative in the way of conceiving the entities of geometry and mathematics. In parallel, Cantor also developed a theory of transfinite ordinal numbers, building an arithmetic for these entities as well.

8. The calculation of probabilities

Another branch of mathematics which had a significant development during the XIX century was the calculus of probabilities. Laplace's work, which in a certain way marks the beginning of a systematic construction of this branch of mathematics, in other respects closes a period of evolution during which the calculation of probabilities had also been cultivated by illustrious mathematicians; which had highlighted his connections with the problems of inductive logic and statistics. Laplace's analyzes retain their validity even if today the concept of probability is more

[24] Cantor Georg (St. Petersburg 1845 - Halle 1918), German mathematician and logician. The family, of Jewish origin, moved to Frankfurt in 1856; Cantor studied in Zurich and then in Berlin, where he attended the courses of Karl Theodor Wilhelm Weierstrass and Leopold Kronecker. Appointed as a lecturer in 1869, he worked on number theory and mathematical analysis. From 1872 he taught at the University of Halle. His first studies, concerning the Fourier series, led him to the enunciation of the theory of irrational numbers, fundamental for the development of contemporary mathematics. In his formulation of set theory, on which modern mathematical analysis is based, the concept of number is extended for the first time with the introduction of infinite numbers or, as Cantor himself defined them, transfinite. His studies were instrumental in the subsequent critical investigations of the foundations of mathematics and mathematical logic.

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frequently presented starting from the subjective attitude of an individual who judges his financial commitment (in the broad sense of the term) regarding an event on which he has incomplete information.

9. Conclusion

In the XX century, science experienced an overwhelming pace of growth and expansion, among other things also due to the continuation and accentuation of that intertwining of science and technology. The mathematics of the XX century gave great importance to abstraction and became increasingly interested in the analysis of general structures. The high degree of formal abstraction that had affected analysis, geometry and topology at the beginning of the XX century could not fail to invade algebra as well. A new type of algebra emerges, which is sometimes described as "modern algebra"; the mathematical symbols "x" and "y" no longer necessarily represent unknown numbers; they can today refer to elements of any kind, such as substitutions, geometric figures, matrices, and so on. While it is true, as mathematics scholars claim, that mathematics changed shape between the two world wars, it is equally true that much of the mathematics after World War II represented a radically new starting point, thus heralding a new epoch. The theory of probability and statistics in the twentieth century were linked not only to pure mathematics, but also with one of the most characteristic aspects of our time, namely the increasingly widespread use of electronic calculators. Between 1920 and 1940, homological algebra was born; it is a development of abstract algebra, and consists in the study of properties common to several different types of spaces; it is an invasion of topological algebra in the domain of pure algebra. In these years we also witness the development of geometry, even if the geometric language does not lose respect to the past, its suggestive force and its heuristic capacity. On the contrary, it remains a precious guide where "analysis" alone would be insufficient to prevent the researcher from getting lost in a labyrinth of notions perhaps defined in a very rigorous way but too little "intuitive". But the twentieth century also brings with it developments in other branches of mathematics; we find innovations in functional analysis and in the renewal of analytical research, in terms of the development of calculation tools there is the spread and improvement of calculating machines, the development of information technology, the development of cybernetics; in short, in this century the development of mathematics plays the part of the "lion". In conclusion, it must be stated that it is true that the twentieth century gave history a considerable contribution of scientific discoveries, but we must not forget, however, that the "rebirth of mathematics" must be attributed to the great mathematicians of the XIX century.

Bibliography

1. Avital S.: History of Mathematics Can Help Improve Instruction and Learning', in F. Swetz, J. Fauvel, O. Bekken, B. Johansson & V. Katz (eds.), Learn from the Masters, The Mathematical Association of America, Washington, DC, pp. 3–12,1995.
2. Barwell M.: The Advisability of Including some Instruction in the School Course on the History of Mathematics, The Mathematical Gazette7, 72–79, 1913

3. Bochenski I.: M.A History of Formal Logic, trans. I. Thomas (Notre Dame, IN: University of Notre Dame Press, 1961.
4. Bortolotti E. Prefazione e analisi di "L'Algebra". In Bombelli, pag. XXV-LIX, 1966.
5. Bottazzini U. - Freguglia P.- Toti Rigatelli, L.: Fonti per la storia della matematica, Sansoni, Firenze, 1992.
6. Bourbaki N.: Elementi di storia della matematica, Feltrinelli, Milano, 1963.
7. Bourbaki N.: Elements of the history of mathematics. Springer-Verlag, Berlin, Heidelberg, ecc., 1994.
8. Boyer C. B.: A History of Mathematics, Princeton University Press, Princeton, 1985.
9. Boyer C. B.: Storia della matematica, Milano, Oscar Mondadori, 1980.
10. Boyer C. B.: History of Analytic Geometry New York: Scripta Mathematica, 1956.
11. Burton D.: M.: The History of Mathematics. An Introduction, 6th ed. New York: McGraw-Hill, 2007.
12. Cooke R.: The History of Mathematics: A Brief Course, 2nd ed. Hoboken, NJ: Wiley-Interscience, 2005.
13. Dauben J. W. Mathematics: an historian's perspective. Pages 1–13, 1994.
14. Elton G. R.: The Practice of History, The Fontanta Library, London and Glasgow in association with Sydney University Press, 1969. Fauvel J. - Gray, J.: The History of Mathematics: A Reader, The Open University, London, 1987.
15. Frajese A.: Storia della matematica ed insegnamento medio, Bollettino dell’Unione Matematica Italiana, III, 337-342, 1950.
16. Goldstine H. H.: A History of Mathematics: From Mesopotamia to Modernity Oxford, New York, 2005.
17. Goldstine H. H.: A History of Numerical Analysis from the 16th through the 19thCentury New York: Springer-Verlag, 1977
18. Goldstine H. H.: A History of the Calculus of Variations from the 17th through the 19th Century New York: Springer-Verlag, 1977
19. Hessenbruch Arne: Reader’s Guide to the History of Science. Routledge, 2000.
20. Katz V. J.: History of Mathematics: An Introduction, 3rd ed. Boston: Addison-Wesley, 2009.
21. Kline M.: Storia del pensiero matematico. Vol. I, Enaudi Editore, 1991, Torino.
22. Kline M.: Storia del pensiero matematico. Vol. II, Enaudi Editore, 1991, Torino.
23. Lolli G.: Filosofia della Matematica. Eredità del Novecento, Il Mulino, Bologna, 2002.
24. Lori G.: Storia delle matematiche dall’alba delle civiltà al tramonto del secolo XIX, Cisalpino-Goliardica, Milano 1982.
25. Loria G.: Storia delle matematiche Vol I, 1929, Torino.
26. Schubring G.- Karp A: Handbook on History of Mathematics Education, New York, NY, 2014

Vol. 6, No. 03; 2021

ISSN: 2456-3676

27. Smith D. E.History of Mathematics, 2 vols. New York: Dover, 1958.
28. Unguru S.: History of Ancient Mathematics: Some Reflections of the State of the Art', Isis70, 555–565, 1979.
29. Weil A. Teoria dei numeri. Storia e matematica da Hammurabi a Legendre. Einaudi, Torino, 1993.
30. Weil A.: History of mathematics: why and how. In Proceedings of the International Congress of Mathematicians Acad. Sci. Fennica, Helsinki, 1980.