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Abstract

In this work we wanted to highlight the importance that the first great French mathematician Cauchy had in the history of mathematics. He is pioneered the study of analysis, both real and complex, and the theory of permutation groups. He also researched in convergence and divergence of infinite series, differential equations, determinants, probability and mathematical physics. We have reviewed all the phases in the life of the great mathematician, thus highlighting all the salient phases that led him to become a mathematician of considerable importance. Finally we highlighted how the life of the French mathematician ended between contrasts and admiration

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1. The early years

Augustine Louis Cauchy¹, the first great French mathematician who belongs definitively to the modern era, was born in Paris on August 21, 1789, a little less than six weeks after the storming of the Bastille. His life and character are as moving as those of poor Don Quixote; in fact, he grew up in an environment of misery and deprivation¹, and survived only thanks to the forethought and common sense of his father Luigi Francesco, who was a model of virtue and piety, two excellent qualities, in which, however, one must not exaggerate. He was a parliamentary jurist, as well as a cultured man, learned in classical and biblical subjects and a practicing Catholic; he was even a police lieutenant at the time of the fall of the Bastille. Two years before the Revolution, he had married Maria Maddalena Desestre, a very good woman, not very intelligent but bigoted like him. Agostino was the eldest of six children, two boys and four girls. It is only natural that, with such parents, Cauchy would become the stubborn Don Quixote of French Catholicism in the 1830s and 1840s when the Church was on the defensive. He suffered for religion and for this he deserves respect, but it must be said that sometimes he deserved to suffer; his eternal sermons on the beauty of holiness did not amuse his neighbor and aroused undeserved opposition to his holy purposes. Abel² who was also the son of a pastor and an honest Christian, expressed the indignation that certain attitudes of Cauchy aroused, writing about him: <Cauchy is bigoted, which is strange in a scientist>. Two great mathematicians, who were endowed with a very noble character, Weierstrass and Hermite, were also Catholic, devout, but not bigoted. Cauchy's childhood took place in the middle of the Revolution; the schools were

¹ In a letter from that period, Cauchy writes: <..... I would like at least some biscuits to eat with my brothers, but this is not possible, because the only food available we have comes from the garden that our father grows>.

² Niels Henrik Abel (1802 -1829) was a Norwegian mathematician, best known for his fundamental contributions to algebra and the theory of functions; also remembered for the award that bears his name.

closed because for the moment, not needing either science or culture, the Commune (the body responsible for governing the city) left scientists and men caught in misery or sent them to the guillotine. To escape the danger, Cauchy the father led the family to his village, Arcueil, where he let the Terror pass, almost starving and feeding his wife and children the few fruits and vegetables he could grow; it turned out that Cauchy was a delicate and physically underdeveloped child; he was nearly twenty when he began to recover from the poor diet he suffered in childhood and had to monitor his health throughout his life. The retreat in Arcueil, which gradually became less hard over time, lasted about eleven years, during which the father took care of the education of the children. He himself wrote school texts, often in easy verse, in which he excelled because he thought that poetry makes grammar, history and above all morality more entertaining and easier to assimilate for the minds of children. For this reason a poetic vein developed in the young Cauchy which he cultivated throughout his life, writing in French and Latin verses that abound with noble sentiments expressed in dignified form, and wonderfully reflect his unblemished life, but have no other merit. . Much of Cauchy Sr.'s lessons were devoted to intense religious instruction, in which his wife helped him. Arcueil was close to the superb domain of the Marquis of Laplace and Count Claudio Luigi Berthollet³, a valuable chemist and original man. The two gentlemen were great friends; their gardens were separated by a wall common to the two properties in which a door opened, to which each had a key. Although the mathematician and chemist were far from devoted, Cauchy Sr. did all he could to get acquainted with his eminent and well-fed neighbors. Berthollet never went anywhere. Laplace⁴ was more sociable and ended up going to the neighbor's house, where he was struck by the sight of the young Cauchy, too physically weak to indulge in the hobbies of his age, immersed in books and notebooks with a monk's air, which, on the other hand, he seemed to like it. In a short time Laplace realized that the boy had a great disposition for mathematics and advised him to take care of himself and not get tired. Laplace's premonition was real; in fact, a few years later, Laplace himself followed Cauchy's lectures on infinite series with some apprehension, fearing that the young man's audacious discoveries would end up upsetting the vast edifice of celestial mechanics, and we were close to indeed that the disaster would happen

³ Berthollet Claude-Louis (1748-182) french chemist. He studied medicine in Turin and in 1772 he moved to Paris where he devoted himself mainly to chemistry of which he addressed, in the course of his work, various sectors. In 1785, he proposed the use of chlorine as a bleaching agent. He collaborated with Lavoisier and others and, in 1787, contributed to the formulation of a new system of nomenclature which forms the basis of the system in use today. He also conducted important research in the field of explosives chemistry and iron metallurgy. In his important work, *Essai de Statique chimique* (1803), Berthollet expounded the theories on chemical affinity and the reversibility of reactions and introduced the notion of chemical equilibrium. In 1780 he became a member of the French Academy of Sciences and, in 1794, professor at the Ecole Normale Supérieure in Paris. In 1798, during his mission to Egypt, it was following Napoleon who appointed him senator in 1804 and, under the empire, count. who saved his head during the Terror because he knew the secrets of cannon powder

⁴ Pierre-Simon Laplace, Marquis de Laplace (1749-1827), was a mathematician, physicist, astronomer and noble French. He was one of the leading scientists of the Napoleonic period, appointed Minister of the Interior in 1799 by Napoleon, who in 1806 conferred on him the title of Count of the Empire, then also appointed Marquis in 1817, after the restoration of the Bourbons. He made fundamental contributions in various fields of mathematics, astronomy and probability theory and was one of the most influential scientists of his time, also for his contribution to the affirmation of determinism. He put the final twist on mathematical astronomy, summarizing and extending the work of his predecessors in his five-volume work *Mécanique Céleste* (Celestial Mechanics) (1799-1825). This masterpiece transformed the geometric study of mechanics developed by Newton into one based on mathematical analysis

and that the "world system" collapsed. If the nearly circular orbit of the Earth had been slightly more elliptical, the infinite series on which Laplace had based his calculations would have diverged; fortunately, his astronomical intuition had saved him from this disaster and he sighed with relief when he had verified the convergence of all his series with Cauchy methods. On 1 January 1800, Cauchy Sr., who had discreetly kept some reports in Paris, was appointed secretary of the Senate and had his cabinet in the Luxembourg Palace, where he reserved a corner for his son as a study room. Since Lagrange⁵, then a professor at the Polytechnic, often visited the secretary Cauchy, he had the opportunity to see his son and, like Laplace, was struck by the boy's disposition for mathematics; one day, in the presence of Laplace and several other personalities, Lagrange, pointing to the little scholar who worked in his corner, declared: <Do you see it? well, I tell you that as a mathematician he will supplant us all>. Lagrange gave some good advice to Cauchy's father, fearing that the boy, of very delicate health, would tire himself too much: "Don't let him touch a mathematics book (he meant, higher mathematics) before the age of seventeen". And another time he said: <If you do not hurry to give Augustine a solid literary education, his inclinations will later and forever remove him from literary studies, and he will be a great mathematician, but he will not know how to write correctly the own language ". The father followed the advice of the greatest mathematician of the time and gave his son a serious literary education before leaving him free to pursue his vocation for mathematics.

2. The first studies

After his father had done all he could for him, at the age of thirteen, Cauchy entered the Central School of the Pantheon; in this school, Napoleon had instituted several prizes and a general competition among the pupils enrolled in the same class of all the schools in France. From the beginning, Cauchy was the ace of the school and reported all the first prizes in Greek, in Latin composition and in Latin verse. When, in 1804, he left the school he had won both the general competition announced by Napoleon among all the French schools, and a special prize in literature. The same year, he made his first communion, which is a beautiful and solemn occasion in the life of a Catholic, and doubly in that of Cauchy. He spent the following ten months working ardently in mathematics with a good professor, and in 1805, at the age of sixteen, he was admitted to the Polytechnic, finishing second in the standings; it cannot be said that he spent precisely happy days there as his companions did not give him peace and made fun of him for his religious practices, but Cauchy always managed to master himself and even tried to convert some of his tormentors. Numerous were the scholastic successes of the young Cauchy; moreover, while he was at the Polytechnic School he devised a new analytical solution to the problem of Apollonius, a solution which, having been given to the press, was then reproduced in

⁵Joseph-Louis Lagrange, (1736-1813), was an Italian mathematician and astronomer active, in his scientific maturity, for twenty-one years in Berlin and twenty-six in Paris. He is unanimously considered one of the greatest and most influential European mathematicians of the eighteenth century. His most important work is the *Mécanique analytique*, published in 1788, with which rational mechanics was conventionally born. In mathematics, he is remembered for his contributions to number theory, for being one of the founders of the calculus of variations, deducing it in the "*Mécanique*", for having outlined the foundations of rational mechanics, in the formulation known as Lagrangian mechanics, for the results in the field of differential equations and for being one of the pioneers of group theory. In the field of celestial mechanics he conducted research on the phenomenon of lunar libration and, later, on the movements of the satellites of the planet Jupiter; he investigated the problem of the three bodies and their dynamic equilibrium with the rigor of mathematical calculation. His pupils were Jean-Baptiste Joseph Fourier, Giovanni Plana and Siméon-Denis Poisson

many French works. We must give him the merit of having been able to use the abilities of others wherever he could discover them. Later in 1807, Cauchy entered the School of Civil Engineers (Ponts et Chaussées); despite his eighteen years, he immediately showed himself superior to his companions who were about twenty years old and had been attending school for two years already. Cauchy's innumerable skills and his daring originality were immediately noticed by the professors of the school of civil engineers and all this allowed him to forge ahead in the usual government careers, even at the cost of overtaking his older colleagues.

3. Cherbourg

After the engineering school (March 1810), he immediately received an important task from the government, namely the direction of the works for the construction of the port of Cherbourg. Napoleon's government in those years had to build a port in Cherbourg, which would have represented an excellent link for French ships and not only during the war for military operations. In fact, according to Napoleonic plans, the port was also to serve as a communication route for trade with other nations. To carry out this great project, Napoleon needed a man with great technical skills, and therefore he turned to the engineering school to send an engineer to Cherbourg capable of directing the work on the port. After a detailed analysis of Napoleon's request, the superiors of the engineering school, having immediately had a high esteem for the young Cauchy, decided to send him to Cherbourg to direct the work of the port. When Cauchy left Paris to reach his first job in Cherbourg, he had little baggage, but high hopes. The Battle of Waterloo (June 18, 1815) was hidden by the impenetrable mists of the future, and Napoleon firmly set out to take England by the throat and land it. But to invade the island, it was necessary to have a very powerful fleet and therefore to begin by building ports and fortifications intended to protect the construction sites against the incursions of English ships. For several reasons, therefore, Cherbourg was the point from which all the grandiose operations of a marvelous project had to logically begin, destined to hasten <le jour de gloire> that the French never tired of singing at the top of their lungs from the storming of the Bastille onwards. Thus the young Cauchy was commanded to Cherbourg, to become a great military engineer. In his light baggage there were only four books: Laplace's Celestial Mechanics, Lagrange's Treatise on Analytical Functions, Thomas da Kempis's Imitation of Jesus Christ and a copy of Virgil's works, an extraordinary assortment for a young and ambitious budding military engineer. Lagrange's treatise was to be the book by which its author's prophecy would be fulfilled; in fact it was precisely this book that prompted Cauchy to seek a theory of functions free from the manifest errors present in Lagrange's. The imitation of Jesus Christ caused him some displeasure because the application of the precepts of the book and the aggressive devotion of Cauchy hurt the nerves of his companions, more eager than him to achieve the purpose of their work, that is to kill others. Cauchy resisted their sarcasm and turned the other cheek, according to the suggestions in the book, but the companions did not disarm and laughed assured him that his devotion would not last long. Cauchy retorted by asking gently what they found wrong in his conduct, because if there was any harm it was his intention to correct it; the answer of young people to such an angelic question is not known. From the historical information gathered, it seems that Cauchy's mother had received disturbing information from a dear friend of the French mathematician,

according to which her dear son had become incredulous; the mother was worried and did not hesitate to write to her son to ask him for clarification on the reasons that led him to abandon his faith in Christ and to choose the path of unbelief. Cauchy, did not hesitate to reply to his mother, sending her a letter long enough and full of more feelings, capable of calming any mother who had sent her child to the front or to any other place of perdition. He reassured his mother, reiterating that his faith in Christ had not been affected at all and that no event in the course of his life would ever be able to affect her; the conclusion of this letter shows that the pious Cauchy was perfectly capable of standing up to his executioners, although they insinuated that the devotion of their comrade was no longer the same as before. Cauchy remained about three years in Cherbourg, during which time he worked incessantly. In a letter dated July 3, 1811, he describes his life without rest and without distractions, and writes: «I get up at four in the morning and I am busy until the evening. This month, my regular work increased due to the arrival of Spanish prisoners, of which we were only notified a week earlier. In eight days we had to build barracks and prepare camp beds for 1200 men. Finally our prisoners are housed and covered; they have field beds, straw, food, and they consider themselves particularly lucky work does not tire me, on the contrary, it strengthens me, so much so that I feel perfectly well ». In addition to this hard work "pour la gloire de la belle France", Cauchy, did the work inherent to his research. From December 1810, he had begun to review all branches of mathematics, from arithmetic to astronomy, clearing up the obscure points, applying his methods to the simplification of proofs and the discovery of new propositions. As if all this were not enough, this tireless worker found the time to give lessons to some colleagues to facilitate their promotion, and even helped the mayor of Cherbourg to organize the school exams; thus, he learned to teach. Finally, he still found some time for his favorite occupations. Meanwhile, the Moscow defeat of 1812, the war against Prussia and Austria and the defeat suffered in Leipzig in October 1813, distracted Napoleon from his dream of invading England; therefore the works of Cherbourg languished and Cauchy decided to return to Paris

4. The return to Paris

Cauchy returned to Paris in 1813, exhausted from excessive work, he was then twenty-four years old and already his interesting researches, in particular his study of polyhedra and that of symmetric functions, had attracted the attention of the most eminent French mathematicians to him. His 1814 study of definite integrals, with limits of complex numbers, ushered in his fine career as an independent creator; no one has equaled it in the theory of functions of a complex variable, of which Gauss had established the fundamental theorem in 1811, three years before Cauchy. Cauchy's study, very detailed, it was published only in 1827; this delay is explained by the length of the work, of about 180 pages; Cauchy had no qualms when bombarding the Academy or Polytechnic with voluminous works ranging from 80 to 300 pages, the expenses of which had to come out of their meager budgets. The following year, 1815, Cauchy caused a bit of a revolution in scientific circles, proving one of the great theorems that Fermat had handed down to posterity: every positive integer is a sum of three "triangles", of four "squares", of five "pentagons", of six "hexagons", and so on, considering zero, in each of these cases, as a number of the category. A "triangle" is one of the numbers 0, 1, 3, 6, 10,15,21 etc, obtained by

constructing equilateral triangles with points: the squares they are built in the same way. So too, the pentagons are regular pentagons constructed by means of points, and so on for hexagons, etc. It was not an easy thing to prove; in fact, Euler, Lagrange, Legendre did not succeed; Gauss had previously given the proof for triangles. As if to show that he was not limited to first-rate works in pure mathematics, Cauchy won the grand prize offered by the Academy for a "Theory of Wave Propagation on the Surface of a Heavy Fluid of Indefinite Depth"; ocean waves are very similar to this type from the point of view of their mathematical treatment. This study took up more than three hundred pages of print. In 1816, Cauchy, who was then twenty-seven, was a first-rate scientist among living mathematicians. His only rival was the uncommunicative Gauss, twelve years his senior. At the age of twenty-seven, Cauchy, encouraged also by public opinion, asked to obtain a seat at the Academy of Sciences, an absolutely exceptional honor for such a young man; the first vacant seat in the math section would be his, so his friends confidentially asserted. Thus, in 1816, Cauchy was ripe for an election to the academy, and while there were no vacancies at the time, it was expected that soon there would be two, given the age of the occupants: Monge⁶ was seventy, Lazzaro Carnot⁷ sixty-three. It should be noted that when Napoleon seized power in 1796, Carnot was exiled for opposing the dictatorship by declaring: <I am the irrevocable enemy of all kings>. Then, in 1812, after the Russian campaign, in which Napoleon tried to conquer Russian territory to increase his hegemony over Europe, the mathematician Carnot, who was in exile, in order to return to France, offered to enter the army, but on one condition, that of fighting for France and not for Napoleon's French empire. After the Hundred Days⁸ War with the defeat of Napoleon, Louis XVIII ascended to the throne with the restoration of the monarchy; during the political upheaval that followed, the Academy of Sciences was reorganized, and King Louis XVIII, by decree of March 21, 1816, arbitrarily

⁶ Monge Gaspard (1746-1818), French mathematician, considered the founder of descriptive geometry; its name is linked to the method of orthogonal projections for the graphic representation of three-dimensional figures. He studied in Beaune, in Lyon and at the military school of Mézières where, at a later time, he was assigned the chair of mathematics and physics. He helped found the Ecole Polytechnique (1794) and taught descriptive geometry there for over ten years.

⁷ Carnot Lazare-Nicolas-Marguerite (1753-1823), a French politician and military engineer, was a prominent figure during the Revolution. He became a member of the Legislative Assembly in 1791, of the National Convention in 1792 and of the Public Health Committee in 1793; even if he was never awarded the position of commander, in the military field Carnot was an excellent strategist and devised most of the plans that led the French troops to victory in the battles of the years 1792-1795. In 1795 he joined the Directory with the post of Minister of War. He recognized the value of Napoleon and prepared the attack strategy against Austria. His profound knowledge of military tactics earned him the nickname "Organizer of Victory". Faithful to republican ideals, he opposed Napoleon when he proclaimed himself emperor, but relations between the two did not break down; in 1814, when the Russians, Austrians and Prussians invaded France, Carnot placed himself in the service of the emperor and the latter, during the Hundred Days, appointed him Minister of the Interior. With the Restoration, the Bourbons exiled him from the country, and Carnot retired to Germany. He wrote some treatises on mathematical theory and various volumes of military tactics, which promoted the introduction of notable changes in fortification techniques.

⁸ The hundred days represent the last period of the reign of Napoleon I, which lasted from his entry into Paris on March 20, 1815 to his second abdication on the following June 22. After escaping from the island of Elba where he had been forced to retreat, Napoleon landed in Cannes and triumphantly marched on Paris despite the fact that an army commanded by Marshal Ney was sent against him in an attempt to stop him. Arriving in Tulleris on March 20, the day after the king's flight, he immediately found the support of Murat. In the following days a government was formed in which Joseph Fouchè and Lazare Carnot entered. Defeated in Waterloo on June 18, Napoleon was exiled to Sant'Elena.

radiated Monge and Carnot and appointed Cauchy and Bréguet⁹ in their place. Bréguet occupied Carnot's seat without making too much noise, but when the young Cauchy went to sit quietly in Monge's chair, the storm broke out: the expulsion of Monge was considered a baseness, and whoever took advantage of it gave proof to least of lack of delicacy. Obviously, Cauchy was in his own right and in good standing with his conscience. Because he adored the Bourbons, whom he regarded as the direct representatives of heaven sent by God to rule France, even though they were inept fools like Charles X, Cauchy simply fulfilled his duty of loyalty to heaven and France by creeping into the chair of Monge. The absolute devotion he showed shortly after to his king shows that he was sincere and did not obey only selfish purposes. After this first step on the path to glory, the young French mathematician, not yet thirty, was quickly filled with honors and important offices. As for its popularity, Cauchy can be said to have reached a climax at that time. In fact, he was soon appointed professor at the Polytechnic School where, from 1815, he was already repeater of analyzes, and shortly after he entered the College of France and the Sorbonne. He succeeded in everything he did; his activity in mathematics exceeded all limits; sometimes, he delivered two long memoirs a week to the Academy. In addition to his own works, he compiled innumerable reports on the memoirs presented to the Academy by other mathematicians, and still found the time to publish non-stop short communications on almost all branches of pure and applied mathematics. In Europe, he had become more famous than Gauss; students and scientists flocked to his lectures to hear his very clear expositions on the new theories he professed, particularly in analysis and mathematical physics; among his listeners were famous mathematicians from Berlin, Madrid, Petersburg. In the midst of so much work, Cauchy also had an hour to fall in love with Aloisa de Bure, whom he married in 1818 and with whom he lived for almost forty years. She belonged to an old cultured family and was, like him, a fervent Catholic; the spouses had two daughters, who were naturally raised as Cauchy had been raised. Happy in his family and prolific at work, Cauchy suddenly saw the sky of his peaceful existence cloud over when the Revolution of 1830 led to the fall of Charles X of Bordone and the rise to power of the Duke of Orleans. Cauchy, a convinced realist and an extremist Catholic, decided to leave the French capital to avoid having to swear allegiance to the new regime, thus giving up teaching at both the Ecole Polytechnique and the Collège de France. Destiny, who often enjoys himself behind men's backs, had to enjoy it a world the day when he made him leave the chair that belonged to Monge to follow that holy man of the king in exile. Cauchy could not shirk this duty; he had taken a solemn oath of fidelity, and to Cauchy an oath was always an oath, that is, a sacred pledge, even if it had been whispered in the ear of a deaf man.

⁹ Abraham-Louis Breguet (1747 -1823) was a Swiss watchmaker and inventor. Famous and ingenious master watchmaker, he founded the famous Breguet watchmaker in 1775, still active today. Countless were his contributions and inventions in the field of watchmaking, his main but not the only interest. The most famous remains the regulatory system in tourbillon, developed since 1795 and for which he obtained a patent in 1801. He also invented several tools physics and astronomy. It is called the Leonardo da Vinci of watchmaking.

5. From Paris to Turin

At the age of forty, in order not to fail in the oath of loyalty to King Charles X of Bordone Cauchy, he left all his very important positions in the lurch and took the path of voluntary exile. After all, he didn't mind leaving; the bloodied streets of Paris filled him with horror and he was firmly convinced that the good King Charles did not enter that slaughterhouse at all. Leaving his family in Paris, but without resigning as an academic, Cauchy went to Switzerland, where he tried to distract himself by giving scientific lectures and continuing his studies. He did not ask the smallest favor of the exiled king like himself; he did not even know if the latter was aware of his voluntary sacrifice, which he had imposed on matters of principle. He lived in Friborg (Switzerland) for a short period where he thought of founding a Helvetic Academy according to monarchical and religious principles but not finding financial support for his project, he decided to leave Switzerland for Italy. In Italy, at that time, several universities were coming to light, and the rulers of the time donated funds for the advancement of culture. Cauchy, having learned of the prosperous situation in which the world of culture was operating, moved to Italy. He hoped, precisely in light of the historical-economic situation in which the academic world operated, to find here the long-awaited economic support to carry out his coveted project, that is, to found the Helvetic Academy. The French mathematician was certain that he would also find in Italy the possibility of continuing his research. Therefore in the autumn of 1830, Cauchy arrived in Italy and immediately came into contact with the Italian mathematical circles; and first with the Milanese mathematician Gabrio Piola¹⁰ (1791-1850) and later with the mathematician Paolo Ruffini¹¹ in Modena; a great friendship was born between the two mathematicians that grew over time. It is known that between the two mathematicians there had already been an intense correspondence; in a letter sent to Cauchy, Ruffini asked him for an opinion on the proof of his theorem, which is now called Abel-Ruffini. In fact, Ruffini was well aware that the French mathematician, in mathematical circles, played a role of the first order and therefore his favorable opinion on the proof of the theorem would in fact break the silence looming over the discovery of the famous theorem of the Italian mathematician. Cauchy promptly replied by reiterating that his proof was correct (and perhaps for some decades he was the only one to affirm its correctness). Another point touched upon in the correspondence between Cauchy and Ruffini: is represented by the Critical reflections on the calculation of probabilities by Laplace, published by Ruffini in 1821 (also linked in some way with Turin, and with the Marquis Cesare d'Azeglio, the father of Massimo, and with his Friend of Italy, but here the discussion would take

¹⁰ Gabrio Piola Daverio (1794 - 1850) was a mathematician, physicist and teacher Italian. He came from a noble family of Giussano. He studied mathematics and physics at the University of Pavia. He wrote numerous treatises and memoirs on physics, mechanics and mathematics. In particular he studied the behavior of bodies under the action of forces. He linked his name to the nominal voltage tensors called Piola-Kirchhoff.

¹¹ Paolo Ruffini (1765-1822) was a mathematician and physician Italian. He was a pupil of Luigi Fantini, a well-known expert in geometry, and of Paolo Cassiani, professor of analysis. His name is linked to the Abel-Ruffini Theorem (probably conceived in 1803 or 1805), partial demonstration of the algebraic irresolubility of equations of degree higher than the fourth, through the theory of groups, and to the decomposition rule of polynomials. Furthermore, in 1809, he published Ruffini's rule, an algorithm for dividing a polynomial into a variable by a binomial of first degree in the same variable. The algorithm allows to find both the quotient polynomial and the remainder polynomial. It is a simplified algorithm compared to the general one for the division of polynomials. In the philosophical field, he tried to demonstrate the immateriality of the soul.

us too far). Naturally, inspired by the same ethical-religious principles of Ruffini, Cauchy could not but take sides with him against certain views of Laplace, against the application of the calculation of probabilities in the moral field; one cannot, writes Cauchy, "attack history by means of formulas, since by means of theorems we study algebra and the calculus of integrals". In these words we find a Cauchy similar to the one that some years after Stendhal¹² albeit as a partisan, would paint in his *Courrier anglais*. It was perhaps the memory of Paolo Ruffini that drew Cauchy's attention to Modena from the first months of his exile, (perhaps the non-recognition of Louis Philippe by the Duke of Modena also played a role). The same letter sent by Cauchy in October 1831 and addressed to the Tsar in which the mathematician asked for financial help to found an Academy in Friborg was sent from the city of Modena; this request, however, was not accepted by the Tsar; today this letter is kept at the National Academy of Sciences, Letters and Arts of Modena. Financial aid to build the Academy of Mathematics in Friborg, never completed, was promised to Cauchy both by the Archduke Maximilian and by the Duke of Modena Francesco IV. At the end of 1831, King Carlo Alberto, king of Sardinia, learning that the famous Cauchy was unemployed, and under pressure from the Jesuits, offered the French mathematician the chair of professor of mathematical physics in Turin, a chair that Cauchy accepted with pleasure. Shortly after his appointment at the Turin university, he fell seriously ill due to exhaustion and emotions and, to his great pain, he had to abandon his evening work for some time. He therefore took a little vacation, which he dedicated in part to a visit to the Pope, and having recovered completely, he returned to Turin full of enthusiasm and ardor, with the intention of dedicating himself again to his courses and research. . Cauchy quickly learned Italian and held his courses in this language, it was reported to the teaching given by Cauchy at the University of Turin, who had proposed to give his lessons in Latin, but then to do something pleasing to the listeners he had replaced Latin with Italian. After all, it is a difficult task to reconstruct, within what limits the actual teaching of Cauchy in Turin could have been carried out, and what its effectiveness could have been. In other words, precisely reconstructing the various phases of Cauchy's teaching at the University of Turin is not easy; after all, at that time teaching in the universities of the kingdom of Sardinia had to be carried out in a very precarious manner. Regarding the provisions adopted by the Magistrate of the Reform of Studies on October 27, 1831, and published in the "Piedmontese Gazette" on November 3, 1831, we read: "Civil Architecture students will, as in last year, take their course privately in their father's houses. Those domiciled in countries where there are no Architects capable of instructing them will be able to obtain permission from the Provincial Reformer to go to other cities or places in the district where they can have the necessary direction. Students of mathematics proper, as well as the students of the 15 free places of Fine Arts and Philosophy aspiring to become Professors, have the right to go and do their studies in Turin where they will follow the private lessons of the Professors". From this provision it follows that mathematics students were allowed to go to Turin but only to follow the professors' private lessons; instead the students of other faculties

¹² Stendhal Pseudonimo of Henri Beyle (1783-1842), french writer. Trained in the Enlightenment and interpreter of the heroic and idealistic moment of the bourgeoisie between the Revolution and the Empire, he is one of the great masters of psychological realism.

were not even authorized to reside in Turin. This highlights the dramatic situation in which the universities and the students themselves found themselves having to work. Moreover, Turin was not the only Italian university in which, in those days, such provisions were adopted. Furthermore, as regards Cauchy's way of teaching, it has been written that most of Cauchy's early pupils found in their memories "a real honor to have learned from Cauchy the fundamentals of pure mathematics and the foundations of integral calculus." To confirm the assignment that King Charles Albert gave to Cauchy there is a document dated January 5, 1832 and still existing in the State Archives of Turin which documents the assignment that Cauchy had. As proof of Cauchy's presence at the court of King Charles Albert, we find the quotation of the French mathematician Cauchy in the king's autobiographical diaries. A few days after his appointment as professor at the University of Turin, Carlo Alberto on January 16, 1832 wrote this other note in his diary: *< Je recus aujourd la visite de remerciement du célèbre professeur Cochy. Lui ayant fait quelques questions sur les sciences, sur les Universités, il me répondit cinq fois « J'avais pensé que V. M. m'aurait interrogé à ce propos et je me suis préparé par une note à lui répondre . Et chaque fois il sortit alors un mémoire de sa poche, dont il me faisait lecture. Il manifesta des vues qui me paraissent fort sages et que je compte d'approfondir >*. However, in the lively description left to us by Carlo Alberto, there is all the methodical and meticulous Cauchy, which emerges, not to say from his works, from the pages of his biographers. Certainly King Charles Albert was delighted to honor a French exile in Cauchy. But his benevolent attitude towards Cauchy can also be framed in the policy of favoring, from the first years of his reign, the sciences and the letters: they were the same years in which the king decorated scientists and artists with honors reserved for the nobility. and to the army, and sponsored the publication of works, such as the monumental *Théorie du Mouvement de la Lune* del Plana (1832). In the Piedmontese capital, Cauchy had to participate personally in political life, if his name appears in some colorful notes on the ultra-share groups of French exiles that can be read in the diaries of the young Camillo di Cavour.

6. The stay in Prague and the education of the Duke of Burgundy

While he was in Turin, where he was teaching at the university, King Charles X, with the good intention of rewarding his loyal partisan, and not of rendering him a bad service, entrusted him with the education of the Duke of Burgundy. his heir, who was thirteen years old. The job of dry nurse and tutor was the last that Cauchy could wish for; nevertheless he docilely joined Charles in Prague, resigned to courageously carrying the cross of fidelity and loyalty; the following year his whole family moved to Prague. The education of the heir of the Bourbons was not a very simple task; from early morning to late at night, Cauchy had the royal boy at his back. Not only did it have to enter, by love or by force, into that brain of a child marred by a bad education, the notions taught in elementary schools, but Cauchy also had to be careful that the little treasure entrusted to his care was not made badly falling while tripping in the park. It goes without saying that the essential part of Cauchy's teaching consisted of conversations on moral philosophy to which the professor was deeply attached, and it is perhaps fortunate that in the end France decided not to take back the Bourbons to make them her beloved masters. Cauchy explained the ideal reasons that led him to devote himself to the education of the Duke of Bordeaux in one of

his writings entitled: "Quelques mots adressés aux hommes de bon sens et de bonne foi" which was published in Freiburg in 1833; in it the mathematician explained that it was his fidelity to the rigid Bourbon teachings that pushed him to this decision. Certainly the essential reason was the duty he felt to reach his king, to listen to "la voix auguste" as he wrote, that "a pu seule me déterminer à quitter la chaire de Physique mathématique que le roi de Sardaigne avait daigné me confier ». But perhaps it is legitimate to attempt to venture some conjecture about other reasons that may have conspired in the same sense. In the family of the ousted Charles X, the tendencies differed; Charles X did not at all support his daughter-in-law, the Duchess of Berry, who with the help of Charles Albert had attempted in 1832 that expedition to France, which began on the steamship, which was named after Carlo Alberto, which ended in failure most complete, and then ended almost in a tragic event between drama and irony, when it was learned that the widow of the Duke of Berry would have given a brother to Henry V. Just in that period of time Cauchy had made a brief appearance in Paris. It is possible that to induce Cauchy to accept the invitation of Charles X, the desire to show the king his immense fidelity and his fullest willingness to serve him was not extraneous, even if this cost him many sacrifices. Another conjecture to be made is to hypothesize that Cauchy could also fear the possibility of a war against France, ardently desired by the king of Sardinia, and not wanted by Austria. Despite the assiduous presence of his pupil, Cauchy managed as best he could to update his works, escaping to his studio in the rare moments of freedom to jot down a formula or scribble a few paragraphs quickly. His most original work in this period was the memory on the dispersion of light, where Cauchy tried to explain this phenomenon with the hypothesis that light is produced by the vibrations of an elastic body. This work is very important for the history of physics, because it demonstrates the 19th century tendency to explain physical phenomena with mechanical models, instead of simply building an abstract mathematical theory to coordinate the observations made. The current practice of Newton and his successors was thus abandoned, although there have been attempts on their part to explain gravitation mechanically. The theory of the elastic solid has had a long and brilliant success, and even today some Cauchy formulas drawn from his false hypothesis are still used, but the theory itself was abandoned when, as often happens, the improved experimental technique and other new phenomena (in this particular case, abnormal dispersion) no longer accorded with the predictions of the theory itself.

7. The definitive return to Paris

Cauchy escaped to his pupil in 1838; he was then nearly fifty years old. For some time, his friends in Paris had been insisting that he return, and he took the pretext of his parents' golden wedding to say goodbye to King Charles and his court. Thanks to a special dispensation, the members of the Institute (of which the Academy of Sciences did, and still does, part) were not required to take the oath of allegiance to the government, so that Cauchy got his seatback. Since then, his business has only grown; during the last nineteen years of his life, he composed more than five hundred memoirs concerning all branches of mathematics, including mechanics, physics, astronomy, and many of these works had the proportions of long treatises. However, his troubles were not over. A vacant seat remained in the College of France and Cauchy was immediately elected to fill the vacant seat, but there were no dispensations here, and before being

able to profess it was necessary to take an oath of allegiance to the government. Convinced that this usurped the divine rights of his sovereign, Cauchy did not bend and refused to swear, but the Office of Longitudes did not want to give up a mathematician of that mold and Cauchy was elected unanimously. Then began a fun game of rope pulling; Cauchy and the Office of Longitudes pulled on one side and the government on the other. Realizing that they were doing nonsense, the government finally let go, and Cauchy remained in the Office without taking the oath; this challenge to the government was flagrant illegality, if not betrayal, but the mathematician persisted in his attitude; his colleagues in the Office politely embarrassed the government by pretending to ignore his demand to name someone legally, and so for four years Cauchy turned his back on the government and continued his work. One of Cauchy's most important contributions to mathematical astronomy belongs to this period. It was Le Verrier¹³ who, without knowing it, provoked this foray into astronomy by presenting to the academy, in 1840, his memoir on Pallas, a long study full of calculations that required the supervisor in charge to examine it as long as it had taken Le Verrier to draft it; Cauchy volunteered for this thankless accomplishment, but instead of following Le Verrier step by step, he took shortcuts and invented new methods that allowed him to verify the work and develop it in a remarkably short time. The conflict with the government became acute in 1843 when Cauchy was already fifty-four years old. The minister refused to continue to be mocked by the public and asked the Office of Longitudes to proceed with the election of a member to fill the post that Cauchy refused to yield. On the advice of friends, Cauchy set out his case in an open letter, which is truly one of his finest things. Whatever one might think of this fight against Don Quixote in favor of a cause that everyone, except a few partisans of the monarchy, knew was definitively lost, one cannot help but feel respect for the courage of Cauchy who, with dignity and without passion, sets out his case and fights for the freedom of his conscience, an old struggle for freedom of thought, in a more common form now than in those days. In Galileo's time, Cauchy would doubtless have gone up the stake; under Louis-Philippe, he denied any government the right to take an oath of allegiance that violated his conscience, and he suffered for his own courage. His attitude attracted the respect of everyone, even of his enemies, and put the government in an equivocal and not very nice light even in the eyes of its partisans. A short time later, certain stupid repressive measures led to fighting in the streets, to riots, in short, to civil war; Louis-Philippe and his clique were driven out in 1848. One of the first acts of the Provisional Government was to abolish the oath of allegiance; the politicians of that time understood that such oaths are either useless or have no value, thus demonstrating rare common sense among them. In 1852 when Napoleon III seized power, the oath was re-established, but this time Cauchy had won before fighting; he was discreetly informed that he could resume his courses without taking the oath; evidently each of the two parties realized that it was not worth the trouble of

¹³ Le Verrier, Urbain (Saint-Lô 1811-Paris 1877), French astronomer; he studied at the Ecole Polytechnique. He improved the astronomical tables of Mercury, studied the perturbations of the movement of comets and investigated the limits within which the eccentricity and inclination of the orbits of the planets vary. In 1846 he attributed the cause of the perturbations of the motion of Uranus to the existence of a hitherto unknown planet, whose position he theoretically calculated; during the same year, the German astronomer Johann Galle observed the new star which was later called Neptune. For this discovery, Le Verrier received many honors, and in 1854 he became director of the Paris observatory.

getting themselves into trouble; the government did not ask for thanks for his liberality, Cauchy did not present any and resumed his courses as if nothing had happened. From then until the end of his life, he was the glory of the Sorbonne. In the interlude between its official instability and its definitive stability, Cauchy had found the time to break a lance in favor of the Jesuits. It was an old dispute between the state, which required Jesuit teaching to at least submit to its directives, and the Jesuits who defended religious education as the only sound basis of any education. Of course, Cauchy, who shared their ideas, took to the fray with his usual impetus and defended his friends with moving and sincere but not convincing reasons. Whenever Cauchy turned away from mathematics, he substituted feeling for reason. The Crimean War offered Cauchy the last chance to alienate his most loyal colleagues; he indulged in enthusiastic propaganda in favor of a singular enterprise known under the name of <Work of the Schools of the East>. According to its promoters, it was necessary to remedy the unrest of the past and at the same time inflict a double check on Moscow's ambition and Muslim fanaticism, and above all it was preparing the regeneration of peoples subjected to the Koran. In other words, the Crimean War had been the usual bayonet that opens the way to the Cross. Very impressed by the need to replace the Koran that brutalized the faithful with something more human, Cauchy threw himself into this project, which completed and consolidated the work of emancipation so admirably begun by French arms. The Jesuit Council, grateful to Cauchy for his recent intervention in their favor, gave him full powers for the execution of a number of details (including that of fund-raising) in view of the moral regeneration of the peoples asserted by the law of the Koran, for the triumph of the Gospel around the cradle and tomb of Jesus Christ was the only reward that justified the rivers of bloodshed by the Christians of France, England, Russia, Sardinia and the Muslims of Turkey in the Crimean war. Unfortunately, Cauchy's colleagues did not appreciate the fervor he put into good works of this kind and called him a hypocrite, an absolutely unjustified epithet for Cauchy who was one of the most sincere practicing Catholics that ever existed. The most evident result of the work in question was the particularly revolting massacre of May 1850; fortunately Cauchy did not live to see the culmination of his labors

8. A life that ended between contrasts and admiration

Cauchy was an admirable professor and one of the greatest mathematicians who ever lived. He lived modestly and was moderate in everything, except in mathematics and religion, about which he usually lacked common sense; whoever approached him was a candidate for conversion.

One day when William Thomson (Lord Kelvin), who was then twenty-one, went to see him to discuss mathematics, Cauchy spent all his time trying to convert his visitor, who was then a fervent follower of the Scottish Free Church, to Catholicism. Cauchy did not avoid the priority disputes in which his enemies accused him of impropriety and greed. The last year of his life was troubled by a dispute of this kind, where it seems that Cauchy was not exactly right, except that he, with his habitual obstinacy when it came to a matter of principle, challenged his point of view with the sweet firmness that no one could be right about. Another circumstance not conducive to Cauchy's popularity with his colleagues: Rightly or wrongly, Cauchy was accused of voting according to the religious and political views of the person to be elected. His later years

were thus saddened by what he considered his colleagues' lack of understanding of what we have previously mentioned and other such weaknesses. He died in Sceaux (France) almost suddenly on May 23, 1857, in his sixty-eighth year of age. To recover from bronchitis, he had in fact retired to the countryside, but was assaulted by a very strong fever which was fatal. A few hours before his death, he chatted with the archbishop of Paris about certain works of charity that he proposed to do; for all his life, in fact, he had a heart of charity, and even on his deathbed he spent his last thoughts for the most needy His last words to the archbishop were: <Men pass, but works remain>. His rigid religious and political sentiments, in contrast to the prevailing ideas of that time, alienated him a lot of sympathies. However, all were in agreement in appreciating, in addition to the height of ingenuity, the rectitude of his character, the nobility and the purity of his mild soul, in fact he was bitterly regretted, not only by all those who were able to admire his genius, but also by the many who benefited from the works of charity to which he had effectively dedicated himself throughout his life. Cauchy's part in modern mathematics is undoubtedly a prominent part, as is universally admitted, albeit sometimes reluctantly. Cauchy was in fact severely criticized for his production which was called "super production", often too hasty: his work includes 789 memoirs (some of which are very voluminous). A criticism of this kind appears unjustified for a man who has produced a large quantity of first-rate works to which other minor ones are added, even if it is generally made by those who have produced little, and this "little" is not superior quality as originality. After his death, but especially in the last few decades, his reputation as a mathematician has definitely established itself. The methods he introduced, which inaugurated the first period of the rigorous exactness that distinguishes modern mathematics, his spirit of discovery, and his works have left such a strong and profound imprint in mathematics that nothing, for many years to come, will to delete.

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