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AGOSTINO LUIGI CAUCHY IN ITALY

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Abstract
In this work we wanted to highlight the importance that the first great French mathematician Cauchy had in the history of mathematics italian. We analyzed relations with some Italian mathematicians and his meeting with the Academy of Sciences of Turin. The relations of the mathematician outside the field of mathematics and the influence of Cauchy on his disciples were also examined.

Keywords: Cauchy, History

1. The relationship with Piola and Plana

In the autumn of 1830, Cauchy arrived in Italy and immediately came into contact with the Italian mathematical circles; and in particular with the Milanese mathematician Gabrio Piola[1]Certainly the relationship between Piola and Cauchy was not immediately one of admiration; in fact Piola initially did not miss an opportunity to criticize the studies of the French mathematician. In some of his writings he argued that Cauchy's arguments did not represent any step forward on the path of exactness and rigor, on the contrary: applying the considerations of limits and last reasons, it is, if we are not mistaken, a retreat that makes on one point who at the same time advances on other points with findings worthy of high praise lingered to discuss what had long ago been decided elsewhere. However, Piola's attitude towards Cauchy's studies soon changed radically; that is, there was a real metamorphosis in the attitude of the Italian mathematician. From being the main opponent of Cauchy's theories, Piola later became one of the greatest advocates. This change, however, took place in stages; from the historical writings preserved at the National Academy of Sciences, Letters and Arts of Modena, there is a letter dated at the end of 1830, which Piola sent to his friend Antonio Bordoni (1788-1860), a mathematician of considerable importance, to warn him of his arrival in Pavia di Cauchy; in it we read: <The famous mathematician Cauchy comes to Pavia accompanied by my friend Frisiani[2]. I would have accompanied him myself if from this race I had not had to fear a shock to

[1] Gabrio Piola Daverio (1794 - 1850) was a mathematician, physicist and teacher Italian. He came from a noble family of Giussano. He studied mathematics and physics at the University of Pavia. He wrote numerous treatises and memoirs on physics, mechanics and mathematics. In particular he studied the behavior of bodies under the action of forces. He linked his name to the nominal voltage tensors called Piola-Kirchhoff.

[2] Paolo Frisiani (1797-1880) Of eclectic training, at the age of twenty-six he began attending the astronomical observatory of Brera where, under the guidance of Barnaba Oriani and Francesco Carlini, he trained as an astronomer. Here he remained as a second astronomer until 1859. In this long period he devoted himself not only to the observation and examination of celestial bodies, but also to mathematical and physical studies. Among his most important works in astronomy we remember: Description of a machine for the movement of a large reflecting telescope, in Giornale dell'Ist. Lombard of Sciences, Letters and Arts, III (1852), pp. 454-459; New photometric apparatuses, in Memories of the Ist. Lombard of Sciences, Letters and Arts, VII (1858), pp. 389-408; On binocular vision, ibid., Pp. 409-411; Helioscopes and celestial photography, ibid., Pp. 411-420; Rectification of astronomical machines with a method independent of plumb line and level, in Italian Library, LVI (1859), pp. 386-395; On the polar auroras, in Acts of the Ist. Lombard of Sciences, Letters and Arts, III (1862), pp. 162 s. In addition to astronomy, F. also made significant contributions to mathematical analysis: Analysis of some transcendent equations, in Astronomical Ephemerides of the Brera

Vol. 6, No. 03; 2021

ISSN: 2456-3676

my health which is always unstable. This distinguished man is forced to leave France because of his principles of true, righteousness and in a few days he is going to Switzerland. I recommend that you show him the university well: after all I won't add a word, since the same reasons why this man is highly respectable for me make him a story for you too and you know well how such subjects must be received>. Contrary to Piola, an amateur mathematician in the truest sense of the term, who accompanied his passion for science to the less intense one for philosophical-religious studies, Bordoni was first and foremost a professor, whose research activity was substantially oriented by that didactics and also many of his memoirs, in addition to his numerous treatises, find their origin and motivation in the problems posed by teaching, if not by the need to provide texts to students. When Cauchy visited Pavia in November 1830, Bordoni was working on a treatise that collected his lessons on the "sublime calculation" and in this activity he availed himself of the help of his friend Piola. Cauchy missed no opportunity to criticize Bordoni's work; who sent, on 6 December 1830, a letter to his friend Piola highlighting the arrogance with which Cauchy had criticized his work. Piola replied to his friend Bordoni and wrote: *"... I am very sorry that you write to me about Cauchy but it does not surprise me, since the works clearly show that he believes himself to be the highest, and since he is not recognized by those who know less than him, so he gets restless and maybe he will finish like others, by being pitied with our utmost regret, or at least regret…".* To testify to the continued admiration for the French mathematician, there is a subsequent letter that Piola sent to the mathematician Bordoni, which denotes precisely the metamorphosis of Piola's attitude towards Cauchy. In this letter dated August 16, 1831, the Italian mathematician highlights the genius of Cauchy, and in it we read: "*This is where I break my head scrolling through the mechanical things of Cauchy scattered in his Exercises, and I feel like suffocating drowned in a deluge of infinitesimals. This author is a great mystery to me! pia that studying becomes an increasingly inexplicable phenomenon to me. Some of his things are so mean, small, ugly and I will also say (but shut up for heaven's sake) wrong, that certainly you, and maybe I too, had a blush that they were printed with our name. At the same time he sends flashes of sublime and inventor ingenuity to many places, which he captivates with wonder. How can these two extremes be reconciled? I have seen something similar between poets: but among mathematicians this fact is really new to me*". In particular, Piola da Cauchy's dissent ends up focusing on mechanical things. In fact, the letter continues: *"All these mechanical things as far as I have seen so far are very easily attached here and there to [Lagrange's] Analytical Mechanics, and with a couple of memoirs that served as a supplement to Lagrange's work they all embrace in a little bit. Maybe, if God gives me life and health, I'll try, but you can do this job much better than me…".* In fact, in a long reply of 7 September, Bordoni, after observing that Piola's opinion on Cauchy's studies was correct; he

Astronomical Observatory, 1845, pp. 3-127; Genesis of symmetric and alternating functions, ibid., 1846, pp. 97-262; On the integration of first-order and linear ordinary differential equations between any number of variables, ibid., 1850, pp. 1-186. Of particular interest are his works on terrestrial magnetism, in part stimulated by the participation of the Brera Observatory in the international program of geomagnetic measurements launched by K.F. Gauss, inspired by which F. wrote Researches on terrestrial magnetism, in Memories of the Inst. Lombard of Sciences, Letters and Arts, VIII (1860-62), pp. 107-201, 485-518; IX (1863), pp. 69-80; XIII (1874), pp. 189-212. He was not only interested in scientific problems, but he also devoted study and attention to other themes: from didactic ones (On the popular teaching of political economy, ibid., VII [1861], p. 441) to those concerning human physiology (On the popular teaching of political economy, ibid., VII [1861], p. 441) On the association of the vibratory points of the nervous system, in Rendiconti of the R. Lombard Institute of Sciences and Letters, VIII [1875], p. 15). In 1875 he left an annuity in favor of the Brera Observatory of 500 lire.

Vol. 6, No. 03; 2021

ISSN: 2456-3676

asked him: *"... How can the Cauchy results be obtained with the derivative method?..."*. However with the passage of time it is still Piola, who is the most attentive reader of Cauchy, who takes up the previous works of the French mathematician and studies those that Cauchy was publishing lithographed at the time: the memoirs presented at the Turin Academy starting from the fundamental one. on "a new calculus, called the calculus of limits" which contains several of Cauchy's most significant contributions to complex analysis, from "Cauchy's formula, to the method of majorants. In a letter dated November 12, 1832, Bordoni replies to his friend Piola who incessantly talks to him about Cauchy and his innovations in the field of mathematics, and in this regard he tried to convince Bordoni of what he said about Cauchy's works. Bordoni in this letter writes to Piola and tells him: *"Well, get me the following works by Cauchy, namely le Lecons sur l'application àl la géomètrie, Lecons sur le calcul différentiel, Memoires sur l'application du calcul des residues à la mécanique ... : suggestions ..."*. In the same year Piola gives life to a magazine, the mathematical and physical booklets of various authors. In other words, Piola through his magazine certainly contributed significantly to spreading Cauchy's ideas and techniques among Italian mathematicians, despite the initial attitude of rejection. Later, together with his friend Frisiani, Piola translated Cauchy's great memoir on the "calculation of limits", enriching it with explanatory notes; in addition, he himself wrote many articles of a compilation nature, on the computation of integrals and on that of residuals, to familiarize the reader with "the modern analysis" proposed by the French mathematician. Only in the field of mechanics Piola remained substantially faithful to the Lagrangian methods, the "generality" of which, he wrote: *"it is a very strong reason to induce to prefer them to other more particular ones"*. However, it should not be forgotten that Piola shared Cauchy's views on religion and science. In the pamphlet of a philosophical-religious nature "Letters of Evasius to Uranium around the mathematical sciences" which appeared first in a periodical and then as a volume in 1825, Piola advocated an apologetic conception of mathematics, to be opposed to the detractors of the Catholic faith and to the supporters of Laplacian materialism. On the other hand, it will be Piola himself who translated into Italian Cauchy's pamplet "A few words addressed to men of common sense and good faith" (Modena 1834), in which the French mathematician, lashing out against "the abuse of and of science used to corrupt hearts and pervert minds "he argued that" the most important interest of the sciences, not excepting those that seem most alien to Religion, consists in joining like so many branches to the divine tree, which is the only one who can give them life and fruitfulness ". The welcome that the French mathematician had found in Turin at Plana[3] was no better. He was professor of astronomy at the University of Turin, and director of

[3] Giovanni Antonio Amedeo Plana (1781 - 1864) was a mathematician, astronomer, surveyor and Senator Italian. From 1800 he attended university at the École polytechnique in Paris, with Lagrange (of whom he was the only Italian pupil and later married a niece), Laplace, Legendre and Fourier. From 1811 he taught astronomy at the University of Turin, receiving the chair that had been held by Tommaso Valperga. For many years he also taught Mechanics to the Student Officers of the Application School of Turin. In a book written by General Luigi Gianotti di Giaveno in which he talks about Plana, he remembers him as a sublime scientist, eccentric and grumpy but always fond of his students. In 1815 teaching at the university was suppressed and Plana moved to the chair of calculus, which he held for the rest of his life. In 1820 he was among the winners of a prize issued by the French Academy of Sciences for the construction of lunar tables based exclusively on the law of universal gravitation. In 1831 Plana built the **Universal Mechanical Calendar** which, thanks to an ingenious system of toothed wheels, chains and screws, is able to identify any day from year 1 to year 4000. Not only that, the calendar also provides information regarding lunations and tides. The work, the only one of its kind in the world, is kept in the sacristy of the chapel of the Merchants in Turin and it is a very high precision instrument. He was the founder of the astronomical observatory of Turin (which before him was only an extension of the chair of astronomy), of which he promoted the construction on one of the towers of Palazzo Madama, in Piazza Castello and who directed for more than half century. From 1851 to his death he was president of the Academy of Sciences of Turin. He was also a member of the Academy of XL, the Académie des

Vol. 6, No. 03; 2021

ISSN: 2456-3676

the astronomy observatory in the city of Turin during the years in which Cauchy worked in Turin; he is particularly known for his fundamental work Theory of the motion of the Moon. In his conversations with the Turin astronomer Cauchy he had experienced firsthand how, in his opinion, the methods commonly used in celestial mechanics and astronomy were still not very rigorous. These considerations, together with the extraordinary length and complication of the astronomical calculations, which he proposed to simplify, were the basis of the long memoir presented by him to the Academy of Sciences of Turin in October 1831: Mémoire sur la mécanique célèste et sur un nouveau calcul appelé calcul des limites. Cauchy said on the subject: "I think that geometers and astronomers will attribute some value to my work when they know that I have come to establish, on the development of functions, both explicit and implicit, general and easy to apply principles, by means of which it is possible not to only to rigorously demonstrate the formulas and indicate the conditions of their existence, but also to set the limits of the errors that are committed by neglecting the remainders that must complete the series ». Published in lithography, the memoir had to reveal all its importance in the eyes of Piola, who undertook the translation into Italian, accompanied by explanatoryn Italian, accompanied by explanatory notes, for the «Mathematical and Physical Booklets», the magazine he founded in 1832 and of which only the first two volumes appeared. This is perhaps the first evidence of a personal acquaintance between Cauchy and the Italian mathematicians; in that same period he presented himself to the scientific public of our country with an article which, due to its programmatic content and the fact that it was written in Italian and published in a very widespread but non-specialist magazine such as the "Italian Library ", Was configured as a real manifesto addressed to Italian scientists, on the" modern "concepts under analysis that Cauchy had been openly supporting for about a decade. The occasion for Cauchy's writing had been a review, probably signed by Giuseppe Cossa (one of the compilers of the “Italian Library”), of Cauchy's Esercises de mathematiques of 1826-1829, which appeared in the same journal in June 1830. The reviewer, while expressing himself in terms of praise overall, did not fail to criticize the obscurity of the symbolism and terminology, as well as the heterogeneity of the articles collected in the Exercises; Finally, he added some general observations on the need for the authors to take the utmost care in presenting the preliminaries and basic elements of the calculation and wrote:

".. Around these principles that serve as a basis for calculus, but are not deduced from it, around these half-apparent and half-occult truths, which so much disturb the inertia of those who want to avoid the toil of reasoning ... they must more than never tire the writer: here every negligence is capital, every reticence is suspect, and where he fails in his intent, let him be at least loyal, confess and do not hide his own impotence…". Although not directly addressed to Cauchy, these reflections by the reviewer solicited an immediate response from the French mathematician who, defending himself from accusations of obscurity, invited his critic to indicate the parts and prepositions that seem to him to be controversial; and he went on to say: "I will be all the more ready to take advantage of your observations as that among those who wrote, I am certainly one

Sciences in Paris and the Royal Society of London (from 1827) and Edinburgh (from 1835). He was known as a proud person and did not form real students, although Menabrea and Felice Chiò studied with him. He is buried in the Monumental Cemetery of Turin.

Vol. 6, No. 03; 2021

ISSN: 2456-3676

of those who have most insisted on the need to rigorously demonstrate the formulas and restrict them between right limits ...". At this point, after a long quotation on the geometric rigor of his methods, taken from the introduction to 1 Course d'analyse Cauchy continued: *"... I wanted to keep this rigor of which I had made a law in algebraic analysis differential, in the calculation of variations and in mechanics…."*. From a technical point of view, in this Cauchy paper there is nothing that he had not already published previously; On the other hand, the vigor and clarity with which he presents his views on some key points of the analysis are new for the Italian public. There is no doubt that Cauchy was fully aware of the reactions that such stances could arouse in Italian mathematical circles, strongly linked to the Lagrangian tradition. According to Bordoni, Cauchy in his article had limited himself to having <spoken> about Lagrange without adding anything that was not already known, Piola realized that the example of the function:

$$f(x) = e^{-\frac{1}{x^2}}$$

it showed that Lagrange was certainly wrong, even if, in his opinion, Cauchy's objection did not touch "the substance of the method." From this the great esteem and admiration that the mathematician Piola had towards Cauchy is highlighted once again, and he demonstrated it also on this occasion.

2. Relations with the Turin Academy of Sciences

Cauchy's relations with the Turin Academy of Sciences occupy a place apart, as is clear from the minutes of the Academy itself, albeit through their protocol language. Several times, in fact, Cauchy was invited to read his dissertations at the academy and intervened for the first time in the session of the Academy number 170 of 11 October 1831 specially convened to hear his dissertation and a few months prior to his appointment at the university of Turin. In the register of the minutes of the Physical and Mathematical Sciences Class of the Royal Academy of Sciences of Turin (vol. III, from 6 March 1831 to 5 December 1839) the meeting number 170 of 11 October 1831 is reported and reads: *"... .. said meeting number 170 of 11 October 1831 is chaired by Count Baldo and the Intendant Cibrario, member of the class, is present, among others. The Cav. Cauchy, member of the Institute of France. I1 Cav. Cauchy, invited by the President, reads various passages from a long writing entitled: Mémoire sur la Mécanique céleste. This reading is taken interrupted by some verbal discussions between the author and Cav. Plana. The latter, requested by Mr. Cauchy, recognizes the part of the writing where he, Mr. Plana, is quoted, and where the historical exposition of some previous interviews followed between the two of them is made. Mr. Plana adds that what he, in one of those interviews had said, he also did, and announces two sealed folds delivered by him on the 2nd and 6th of last September, to the Secretary, who produces them. The opening of these sealed folds is judged untimely, and less proper to shorten and clarify the discussions, especially after Mr. Plana, requested by Mr. Cauchy, declares that the point he dealt with in the aforementioned sealed papers, is only one of the many treated by Mr. Cauchy in his memoir, and it is the general development of the function which serves to express the disturbing forces of the planets. Once Mr. Cauchy has finished reading those parts of his work which are capable of it, that is, which are not mere algebraic signs, the President asks the Author for the use he intends to make of his*

Vol. 6, No. 03; 2021

ISSN: 2456-3676

work. Mr. Cauchy says he wants to resign him to our Academy so that it can dispose of it as it usually does on the works presented to it; however, he asks to withdraw his writing for a short time, after the Secretary has signed all the pages". In this session it must have occurred as a clash between Cauchy and Plana: perhaps Plana, an excellent mathematician, but more astronomer than mathematician, in Cauchy's memoir entitled to celestial mechanics and other things, saw celestial mechanics more than other things, which also opened new horizons in the theories of analytic functions. Some trace of those discussions also emerges from the summary of the Memory on Celestial Mechanics, read in that first session and also published in lithography. However, whoever reads the minutes of the Academy receives the impression that, as the months go by, Cav. Cauchy as if he were making himself an antechamber. Certainly none of the Memoirs, however important they were, read by Cauchy at the Turin Academy, was published in the Academic Acts. Cauchy was a member of various Italian Academies, first, in chronological order, that of Modena; in fact he was appointed correspondent of the Royal Academy of Sciences, Letters and Arts of Modena in the session of January 14, 1829. In a letter that Cauchy sent to the Royal Academy of Sciences, Letters and Arts in Modena, the mathematician expresses his gratitude for the honor accorded him with his appointment as a member of the Academy:

« M. Ricciardi secrétaire généra de l'Accadémie royale de Sciences Lettres et Arts. Monsieur, j'ai recu la lettre par laquelle vous voulez bien m'annoncer que 1'Académie Royale des Sciences Lettres et Arts de Modène m'a fait l'honneur de m'admettre au nombre de ses associés correspondants.

Je vous prie de vouloir bien offirir mes remerciments à l'Accademie, et de lui dire combien je me félicite d'etre aggrégé à une société qui renferme dans son sein tant d'illustres savants. Cet honneur, dont je sens tout le prix, sera pour moi un nouveau motif de chercher à contribuer par mes efforts aux progrès des sciences physiques et mathématiques. Permettez moi de saisir avec empressement cette occasion pour vous prier d'agreer l'hommage de ma parfaite considération.

A. L. CAUCHY
membre de 1'Académie des Sciences

Sceaux-Penthièvr, ce 6 aout 1829».

Cauchy, however, never became a member of the Turin Academy of Sciences. In fact, he did not follow a proposal made in 1832 by Saint Réal and reformulated also in 1833; among the numerous new foreign academics (whose nomination had not been made since 1814), Arago, Gauss, GayLussac and others were named among the scientists, but not Cauchy. After all, given things in the form of historical nemesis, perhaps Cauchy's failure to appoint the Turin Academy of Sciences compensates, in a certain sense, for the way he entered the Académie des Sciences in 1816, when Cauchy and Monge and Carnot "purged" of the Restoration (decree of 21 March 1816); so that some of his brothers never forgave him. The mathematical historians who have highlighted in their writings the immense contributions of Cauchy's work in Turin have compensated Cauchy for failing to be appointed to the Turin Academy of Sciences. In this

Vol. 6, No. 03; 2021

ISSN: 2456-3676

regard, a sentence by a mathematical historian of the time who wrote: *"Cauchy was a great mathematician in the Turin Academy of Sciences and she was and is proud of him even though she denied him to become a member, but of this she certainly today deeply regrets the mistake she made ... ..."*.

3. Cauchy's indelible imprint in Turin

Cauchy left an indelible mark in Turin, and his studies are collected in the «MemorieTorinesi» and in the «Turiner Abhandlungen». Cauchy himself often refers to the Turin memoirs in his subsequent production. In those Memoirs, among other things, we find the basis of the theory of analytic functions, no longer in the old sense of Lagrange, but in the current sense: there is the formula of the Cauchy integral, with those consequences that we want to expose. even today, such as Taylor's series developability in a circle inside the field of existence, and the residual theorem in its most general form. There is also the criterion for determining the number of complex zeros of an analytic function contained within a given boundary. Therefore, truly fundamental results belong to the Turin period, even if it is not easy to completely isolate Cauchy's Turin research from the previous and subsequent ones, all the more so because Cauchy wrote a great deal, and returned several times to the same topic. Certainly not to Cauchy could Gauss's motto be applied: "pauca sed matura".

These Turin works are collected in one of the last two volumes of the Ouvre of Cauchy; so for the first time they are accessible, in their original form, to the entire mathematical public, much of which, has heard of them, but in their entirety they have never seen them, because they have had little diffusion. In reality they are Turin only as lithographs, but, in print, they have never been published in Turin. Cauchy himself, in later works, sometimes refers to information on the results he obtained in Turin, which was a rather rare, if not unique, circumstance given to the Gazette de Piémont in 1832. In confirmation of this, we report the extract of a letter sent by Cauchy to Coriolis[4] (now collected in complete form in the comptes rendus of 1837) which reads: *"... Au rest, j'avais déjà donné ce dernier théoreme dans un Mémoire présenté à 1'Académie des Sciences de Turin, le 10 septembre 1832. Première Iettre sur la détermination complète de toutes les racines des éguations de dégré quelcongue....."* In this letter Cauchy continues to speak stating that the formula in question had been discovered in Turin and he says: *" de laquelle j'aivais déduit le théorème II et les autres théorèmes enoncés dans la Gazette de Piémont du 22 septembre 1832..."*. Of extraordinary importance are the following two small paragraphs which appeared in the 1832 "Gazzetta Piemontese" dedicated to Cauchy; it is clear from them the great imprint left by the French mathematician and the great consideration he received in the Turin mathematical circles for his studies. The first article appeared in issue 5 of January 12, 1832, and on page 26 we read: *"An immense step has been taken lately in mathematical analysis; the calculation of the series can be said to have received*

[4] Coriolis (1792-1843), French engineer and mathematician who highlights the "compound centrifugal forces", called "of Coriolis". Engineer of the Bridges and Roads department, already author of important mechanical works which resulted in a theorem bearing his name, in 1835 he demonstrated that a body moving in a rotating frame of reference is subject to a complementary force ("force of Coriolis ") perpendicular to the direction of movement of the body itself in that reference system. Although provided with a low intensity on the earth's surface, this force, generated by the rotation of the planet, affects the direction of sea and air currents and serves to explain, for example, the circular motion of hurricanes.

Vol. 6, No. 03; 2021

ISSN: 2456-3676

its completion, and a great number of questions, which could not be solved by the infinitesimal calculus alone, can now be resolved with the new calculation of the limits of cav. Cauchy" . The second article appeared in n. 113, of 22 September 1832, on p. 620; it reads: *"In number 5 of our Gazzetta this year some mention was made of a memoir that the Knight Agostino Cauchy had presented to the Academy of Sciences of Turin, and in which he carried out a new calculation especially applicable to the problems of celestial mechanics. , to the developments of the series functions and to the assignment of the limits of the errors that are committed at each term of such developments. A second memoir that the Author presented to the prefate Academy on November 27, 1831, contains a suitable method for determining the number of roots which, in a given equation, fill certain conditions. By applying this method to an algebraic equation of which degree one wishes, one immediately determines, by means of a simple algebraic division, the number of real or imaginary roots which offer, for example, real parts included between given limits or modules included between given numbers (the module of two conjugated imaginary roots being nothing more than the square root of their product) and general theorems are established that embrace as particular cases and the rule of signs of Descartes, and the theorems of Abbot Degua, of Mr. Sudan, of Mr. Sturm etc. Today (September 10, 1832) Mr. Cauchy has added to his previous memoirs other researches which we believe are no less useful and worthy of the attention of geometers. These are general forms for solving equations, in particular algebraic ones, of all degrees, and for the rectification of all curves. In order to give an idea of this new work, it is useful to briefly indicate some of the most relevant propositions to which the author has arrived, and which serve as a basis for the formulas that he establishes. An algebraic or transcendent equation that encloses a variable parameter can, for a given real or imaginary value of this, fully solve by pulling it to the equation that would be obtained by reducing the parameter to zero, if the modulus of it is less than the smallest of the modules to which they have the values of the parameter able to make the equation and its derivative verifiable with the same value of the unknown. In any case, the consideration of the values of the parameter that offer this character, and which the author calls its main values, will provide the resolution of the proposal . After all, Mr. Cauchy proves that, even without solving the auxiliary equation capable of providing the principal values of a parameter, it is possible to calculate in general certain roots of the given equation, assuming that the one obtained when suppressing the first or last term is solved. of the proposal, or when the sum of the terms of even degree or the sum of the terms of unequal degree is equal to zero, the proposal being of even degree. In this way, a given equation is immediately replaced by another one of a degree of less than half. It is useful, indeed it is worth noting, that in this direct resolution of the equations, Mr. Cauchy easily determines a limit of the error that occurs when truncating the convergent series that provide the real or imaginary values after a certain number of terms. The very simple rules which he establishes in this regard are partly based on the new formulas which he proposes for the rectification of the curves, and of which too much would distort us from our intention to give them here that account, which would be required by their importance."* Together with the "Memorie Torinesi" we must remember the "Résumeés analytiques", also born in Turin, which are included in the famous collection of Exercices by Cauchy, for the publication of which Carlo Alberto granted him a

Vol. 6, No. 03; 2021

ISSN: 2456-3676

special aid as shown in the copy of said order filed with the Turin State Archives, in the Register no. 70 Control Licenses, dated 26 September 1833 in which we read:

"*In order to encourage the Knight Agostino Cauchy Professor of Sublime Physics at the Royal University of Turin to complete the printing and publication of his work entitled Résumés analytiques, we are willing to order that on the budget of the Internal Economic Company is paid to the same sum of two hundred lire for each file, or dispensation, which will be brought to light, with which however it has to make fifty copies of each file available to our Secretariat of State for internal affairs. However, we hereby have assigned for this object, as we assign, the annual sum of eight hundred lire that we send to the Treasurer of the General Economic Company of the interior to correspond to the aforementioned Knight Agostino Cauchy; at the rate of two hundred lire for each file of the aforementioned work, which will come to light starting from the first of January of the next year one thousand eight hundred and thirty-four, and continuing in the future until the completion of the work itself, with which evidence of the remission made for it of fifty copies of each file to our Royal Secretariat of State above. We send whoever is responsible for observing, and having the present observed, and the Office of General Control to record it, that such is our mind.*

Turin September 21, 1833
Carlo Alberto ».

4. Relationships outside the field of mathematics

Unfortunately, there is little information on the relations established by Cauchy in Turin outside the strictly mathematical field. University documents show the name of the Marquis Del Carretto, the document that testifies the relationship between the French mathematician Cauchy and the Marquis Del Carretto is now kept in the historical archive of the University of Turin in volume VII of the correspondence protocol, in this volume we find the correspondence that goes from June 1833 to December 1833; here is in fact registered, with the number 5666, a letter from the Secretary of the Royal University, dated 8 August 1833, in which we read: <*participates H.E. the Chief President that Mr. Marquis Del Carretto is in charge of what concerns the interests of Cav. Cauchy Professor of Sublime Physics, which Fr. La Chèze of the Society of Jesus is in charge of what belongs to the press of the said Professor and to collect from the Royal Printing House 50 copies of Résumés analytjques files that the Magistrate of the Reformation has decided to purchase through a bon to the Royal Printing House*>. Among the contacts entertained by Cauchy there is also the relationship with a Polish Jesuit father, whose name is Frenchized in Lachèze[5], who taught in Turin in the college of SS. Martyrs. Another important relationship took place between Cauchy and Bidone[6] and as proof of this there is a

[5] It must be Fr Raimundus Lachaise, so mentioned in the Catalogs of the religious Province of the Society of Jesus of Turin, where the Slavic name (Chzerkas, or more often Czerkas) is added in brackets: born on February 28, 1801, entered the Society of Jesus on 14 August 1815, ordained a priest in 1830, in 1831 he became a student of theology in the Roman college. In 1832 it does not appear in the Turin province; in 1833 he taught mathematics and physics to the students of the Company in the Collegio dei SS.Martiri in Turin. In 1834 he appears in the Nice College with the same post; in 1837 at the Collegium Iriense; in 1838 professor in Savoy; then the traces are lost.

[6] Bidone Giorgio (1781-1839), known both as a mathematician and as a hydraulic engineer. From 1815 professor of hydraulics at the University of Turin, he devoted himself to theoretical studies and experiences of hydraulics. Known for having studied the so-called Bidone jump: abrupt rise in level that is formed in a free surface channel, in the section where the passage of fast motion to slow motion occurs

Vol. 6, No. 03; 2021

ISSN: 2456-3676

letter dated in Turin 4 November 1835 (whose minute, together with Cauchy's answer, dated Prague 13 November 1835, is kept in the library of the Academy of sciences of Turin), in which Bidone thanks Cauchy for sending the Memoria sur interpolation, and asks him for a comparative judgment between the method proposed by Cauchy himself and that of least squares, and then added that he is proud to entertain with Cauchy an epistolary relationship. Cauchy in his reply thanked him for the estimate received and sent him his opinion on the question asked in the letter mentioned above. Other names can be found in Valson, the greatest biographer of Cauchy, often more panegyrist than biographer and not always perfectly informed. Among them we find the count of Senfft, who had been Austrian ambassador to Turin, and who on that occasion came into contact with Cauchy. Cauchy went to visit Senfft[7] to ask him to intervene with the Emperor of Austria so that he would grant him a suitable contribution to found the Academy of Mathematics. On that occasion, Cauchy explained that he had already personally sent his letter of request to the Emperor on 7 October 1830 but had not received any response regarding his request. Senfft agreed to help Cauchy and sent a letter to Metternich[8] to persuade the Emperor to subsidize the Cauchy project. However, it is to be believed that Cauchy's question to the Emperor has remained unanswered, since otherwise, we would have found, in the Vienna Archives, the document that would have attested the imperial decision on the request forwarded by Cauchy. In the letter sent by Cauchy to the Austrian emperor on 7 October 1830, to ask him for a contribution to found the Helvetic Academy, we read:
«Sire,

Tandis que la France, se trouve violemment distraite des méditations scientifiques par des évenements extraordinaires, des membres de 1'Institut et des professeurs, que la résolution derester fidèles à leurs serments oblige d'abandonner leurs chaires, se sont demandés si le moment n'était pas venu de faire participer de nouvelles contrées aux avantages que procure l'étude deshautes sciences. Ayant appris que depuis longtemps les Cantons Suisses et particulièrement le canton de Fribourg désiraient jouir de ces avantages, ilsont formé le projet de fonder dans la ville de Fribourg une Académie nouvelle, et d'y ouvrir des cours ou les jeunes gens, francais ou étrangers à la France, viendraient completter l'instruction acquise dans le collège de cette ville ou dans d'autres établissements. I1 est digne d'elle de se placer à la tete des fondateurs d'une Académie dont l'unique but sera de montrer comment les sciences, unies à la religion, peuvent,en perfectionnant I'homme, contribuerà son. Je suis avec respect, Sire, bonheur, de Votre Majesté Impériale et Royale le très humble, très obéissant et très soumis serviteur

Augustin Louis Cauchy».

[7] Senfft was ambassador to Turin during the years in which Cauchy stayed there.
[8] Metternich, Klemens Wenzel Lothar von (Koblenz 1773 - Vienna 1859), Austrian statesman and diplomat, was one of the dominant figures on the European political scene between 1814 and 1848. Coming from an aristocratic family, he attended the Universities of Strasbourg and of Mainz. In 1794 his family moved to Vienna to escape the French revolutionary troops and Metternich met Countess Eleonore Kaunitz, his future wife, who introduced him to court politics. He served for the Habsburgs, first as an envoy to the Rastatt congress (1797) and then as ambassador to Saxony (1801), Prussia (1803) and France (1806).

Vol. 6, No. 03; 2021

ISSN: 2456-3676

In the Piedmontese capital, Cauchy had to personally participate in political life, if his name appearsin some colorful notes on the ultra-share groups of French exiles that can be read in the diaries of the young Camillo Benso di Cavour[9]. It is not known whether Cavour, in his youth, personally knew Cauchy, but he certainly heard about him in the Turin salons, and mentions him, in

a letter from the end of 1832 in which Cavour mentions the French reactionaries, the ultras, who recently arrived in Turin, who, according to him, sing, shout and scream louder than ever and abandon themselves to the inspirations of their divine frown at France, the century, civilization, and even the human race. Well, to describe those ultras, Cavour says that they make Cauchy's "exaggeration" pale.

5. Cauchy's disciples

The beneficial influence exercised by Cauchy both through teaching and through the numerous publications, was such that if we wanted to draw up a list of his disciples we would have to draw up a list of all the analysts who lived after him. Many were those who continued and completed his work, starting from the very point where he had arrived. Among the direct disciples of Cauchy we must also remember the Italian Francesco Faà di Bruno[10], who was born in Alessandria on March 7, 1825, and taught higher analysis at the University of Turin from 1871 until his death on March 26, 1888. In his lectures he expounded Cauchy's discoveries with great fervor and urged his pupils to study his optics works, in which he recognized an unexploited mine. The historians of mathematics affirm that the last Piedmontese pupil of Cauchy in chronological order was Faà di Bruno, then not yet abbot, whom I studied in Paris, between 1849 and 1856, where he graduated with two theses of analysis and of astronomy, which, at least in part, are influenced by Cauchy. But what, apart from mathematics, brings the teacher and the disciple closer together is the profound religious zeal of both, which has guided the attitudes of both also on the political level. Full of meaning in this respect are the biographical notes of

[9] Cavour, Camillo Benso count of Turin (1810-1861), Piedmontese statesman and prime minister of the Kingdom of Italy; he was one of the main protagonists of the Italian Risorgimento. Cavour has collaborated with some Italian, Swiss and French magazines with a series of articles in which he analyzed issues such as pauperism, customs liberalism, railways, the modernization of agriculture, developing the belief that national independence was a historically objective. founded. In Piedmont he began his political activity in the last years of the reign of Carlo Alberto, marked by the experience of the Statute and the liberal reforms, which was followed by the participation of the Kingdom of Sardinia in the first war of independence. Cavour founded the newspaper "Il Risorgimento" with some Piedmontese moderates, which he directed for a year (1847-48), continuing to collaborate until 1850, when he was appointed Minister of Agriculture in the government of Massimo d'Azeglio. After being minister of finance, King Vittorio Emanuele II appointed him head of government (1852), a position that allowed him to take measures for the economic development of Piedmont and for the construction of a railway network.

[10] Francesco Faa di Bruno (1825 - 1888) was an officer, mathematician and priest Italian. After serving in the Savoy army, he became a professor of mathematics at the university and the military academy of Turin. He published important studies on the theories of elimination and invariants and on elliptic functions. Later he was ordained a priest and founded the Opera di Santa Zita, the congregation of Sisters Minime of Nostra Signora del Suffragio is an educational institution in Turin, which today includes nursery, primary and secondary schools. He owes considerable contributions to algebra (elimination theory, binary form theory, etc.) and the invention of astronomical and meteorological interest (the periscope for representing the phases of the moon, etc.). He also devised a writing device for the Czechs. He was beatified by Pope John Paul II on 25 September 1988. Although not canonized, he is considered one of the Turin social saints.

Vol. 6, No. 03; 2021

ISSN: 2456-3676

Cauchy published in 1857 by Faà di Bruno in the newspaper L'Armonia[11] and reproduced in a pamphlet that is almost impossible to find today. Completely unavailable are some letters that Cauchy sent to Faà di Bruno, which still existed towards the end of the last century, and which demonstrated the close friendship that had been born between the two scholars, but unfortunately the letters of these letters have been completely lost. traces, even if the historians of mathematics have not yet given up and continue their research in the hope of one day finding and thus being able to reconstruct in detail the relationship that existed between Cauchy and Bruno's Faà. When Cauchy had already returned to Paris for many years, another Turin mathematician Felice Chiò[12] approached him to submit his research on the Lagrance series, research that had not been well received at the Academy of Sciences. of Turin. The Academy, in fact, had judged the works not worthy of being published. Chiò then turned to Cauchy, who encouraged him to ask for the judgment of the Académie des Sciences, to which Chiò presented his works, which were approved in the meetings of 7 September 1846 and 1 March 1852, being Commissioners Binet[13] and Cauchy (rapporteur). Cauchy did justice to Chiò, and had his Memoirs published.

Chiò's Memoirs were published int. XII, 2nd series, 1853, of the Recueildes Savants étranners. with the title Recherches sur la série de Lagrange. Chiò gave news of Cauchy's favorable judgment in the eighth congress of Italian scientists, in Genoa, in the session of 18 September 1846, as reported by Genocchi[14]. . It was logical that Chiò would solicit Cauchy's opinion not only for the pre-eminent position that Cauchy held among European mathematicians, also because the study elaborated by Chiò centered around a topic that Cauchy had particularly

[11] L'Armonia, a clerical newspaper (whose full title is The harmony of religion with civilization) founded in Turin in 1848 by the priest G.Margotti (who held the direction until 1863) and by others: first biweekly, newspaper in 1855; suspended in 1859, reappeared in 1860 and moved to Florence in 1866, ceased shortly after publication. Nor did the Catholic Unity, founded by Margotti himself in 1863, continue.

[12] **Felice Chiò** (1813 - 1871) was a mathematician and politician Italian. He studied at the University of Turin, where he had Giovanni Plana among his teachers and graduated in philosophy in 1835. Starting from 1854 he taught mathematics at the Military Academy of Turin and mathematical physics at the University. In 1841 one of his memoirs which corrected an oversight of Lagrange regarding the Lagrange series was rejected by the Academy of Sciences of Turin, on the recommendation of Giulio and Menabrea. The content of the memoir, however, was correct, and was published in the Comptes Rendus between 1844 and 1847 by the Académie des Sciences in Paris. In 1846 Chiò communicated these results to Genoa, at the eighth congress of Italian scientists, arousing a long controversy with Menabrea. His other writings speak of the theory of hump curves, of the calculus of finite differences, of determinants and are mostly of a critical nature. His studies have several original results, especially in the area of pure analysis. Also involved in politics, he was a member of the subalpine parliament for six terms.

[13] Jacques Philippe Marie Binet (1786 - 1856) was a mathematician and astronomer French. Binet entered the École polytechnique as a student in 1804 ; After graduating in 1806, he worked for the Ponts et Chaussées department, but the following year he returned to the École polytechnique as a repeater of descriptive geometry. Later he was professor of Mechanics, then inspector of studies. In 1823, he succeeded Jean-Baptiste Delambre in the chair of astronomy at the Collège de France . Like Cauchy, of whom he was a friend, Binet was a staunch Catholic and a supporter of the Bourbon family's claimant to the French throne. The government in July removed him from his duties at the École polytechnique, but he retained his posts at the Collège de France. His works on pure mathematics, mechanics and astronomy are published in the École polytechnique and in the Journal de Liouville. He is responsible for important work on Euler's phi function, on the study of expressions that depend on the law of large numbers, on the fundamental properties of second-degree homofocal surfaces, first discovered by him, on the motions of planets, on equations to finite differences. for which he formulated an interesting theory. His work on matrix calculus led him to the expression of *the* nth term of the Fibonacci sequence. In the field of astronomy, its kinematics formulas give the expression in polar coordinates of the speed and acceleration of bodies subject to central acceleration, such as the planets of the solar system.

[14] Genocchi Angelo (1817-1889), mathematician, jurist and patriot. Professor of Roman law in Piacenza, he was forced to emigrate to Turin, having participated in the revolutionary uprisings in the years 1848-1849. He devoted himself to mathematics and in 1857 became professor of geometry and analysis at the University of Turin. He dealt with different and elevated branches of mathematics (from the theory of numbers to that of series, to that of elliptic and abelian integrals, etc.), and also with the critique of foundations and the history of mathematics.

Vol. 6, No. 03; 2021

ISSN: 2456-3676

studied, and which indeed according to Brill[15] and Noether[16] had been for Cauchy a starting point towards some of his fundamental results.

Bibliography

1. Belhoste B.: Cauchy. Un mathématicien légitimiste au XIXe siècle (Paris, 1985).
2. Belhoste B.: Augustin-Louis Cauchy. A Biography (New York, 1991).
3. Grabiner J.V. The Origins of Cauchy's Rigorous Calculus (Cambridge, Massachusetts, 1981).
4. Valson C A: La vie et les travaux du baron Cauchy (Paris, 1868).
5. Bogolyubov A. N.: Augustin Cauchy and his contribution to mechanics and physics (Russian), Studies in the history of physics and mechanics, 'Nauka' (Moscow, 1988), 179-201.
6. Cutland N.- Kessler C., E Kopp and D Ross, On Cauchy's notion of infinitesimal, British J. Philos. Sci. 39 (3) (1988), 375-378.
7. Dahan-Dalmedico A.: Les travaux de Cauchy sur les substitutions: étude de son approche du concept de groupe, Arch. Hist. Exact Sci. 23 (4) (1980/81), 279-319.
8. Dahan-Dalmedico A.:, Étude des méthodes et des 'styles' de mathématisation : la science de l'élasticité, in R Rashed (ed.), Sciences à l'Epoque de la Révolution Française. Recherches Historiques (Paris, 1988), 349-442.
9. Dahan-Dalmedico A.: La mathématisation de la théorie de l'élasticité par A L Cauchy et les débats dans la physique mathématique française (1800-1840), Sciences et Techniques en Perspective 9 (1984-85), 1-100.

[15] Brill Alexander (1842-1935), German mathematician. He was a professor from 1884 to 1918 at the University of Tübingen, in his scientific work the mathematician is linked to the addresses of B. Riemann and is with R.Clebsch and M. Noether one of the first scholars of the geometry of curves and algebraic surfaces. His work entitled Uber die algebraischen funktionen und ihre Anwendung in der geometrie (1873) is fundamental.

[16] **Max Noether** (1844-1921) was a German mathematician. His paternal grandfather, Elias Samuel, had started his business in Bruchsal in 1797. Elias had nine children, one of whom was Hertz Samuel. In 1809, the *Edict of Tolerance* was established in Baden-Baden, which assigned a German surname to the head of the family of every Jewish family that did not yet have one. In this way, the Samuel family became the Nöther family, and the name of the son Hertz (Max's father) became Hermann. When he reached eighteen, Hermann moved to Mannheim where, together with his brother Joseph, he started a hardware business. Max Noether was the third of the five children of Hermann and his wife Amalia Würzburger. At the age of fourteen, Max Noether contracted polio, and his entire life was marked by the effects of this disease. Due to his physical problems, he was no longer able to reach school, and for some years he continued his high school studies receiving lessons at home. As a self-taught, he undertook advanced mathematical studies and entered the University of Heidelberg in 1865; here he had Jacob Lüroth and Gustav Kirchhoff as teachers, and in 1868 he obtained a doctorate in astronomy. Subsequently he devoted himself to mathematical research in Gießen with Alfred Clebsch; here also began a long collaboration with Alexander von Brill. In 1870 he obtained a free lecturer at the University of Heidelberg with a thesis on rational curves. In this university he taught first as a free lecturer, and from 1874as an extraordinary professor. In 1875 he moved to the University of Erlangen, where he became full professor in 1888. In 1880 he married Ida Amalia Kaufmann, daughter of a wealthy Jewish merchant. Two years later their first daughter was born, Amalia Emmy Noether, who will become a central figure in abstract algebra. A son named Alfred was born to them in 1883, who later studied chemistry and died in 1918. The third child, Fritz, was born in 1884. Like Emmy, Fritz Noether was also successful as a mathematician. Little is known about their fourth son, Gustav Robert, born in 1889, who suffered from constant illnesses and died in 1928. Max Noether worked at the University of Erlangen for many years, first as an extraordinary professor and from 1888 as a full professor. In Erlangen he died on 13 December 1921. Max Noether was one of the preeminent figures in algebraic geometry of his time. His work was influenced by Niels Abel, Bernhard Riemann, Arthur Cayley and Luigi Cremona; following Cremona, he studied the properties of invariant algebraic varieties for birational transformations. Due to the importance his daughter has achieved, the reference texts available generally describe him as Emmy Noether's father, rather than the other way around (as happens in ordinary families). A family friend, Edmund Landau, described Emmy as "the origin of the coordinate system " for the Noether family.

Vol. 6, No. 03; 2021

ISSN: 2456-3676

10. Dahan-Dalmedico A.: La propagation des ondes en eau profonde et ses développements mathématiques (Poisson, Cauchy, 1815-1825), in The history of modern mathematics II (Boston, MA, 1989), 129-168.
11. Dahan-Dalmedico A.: L'intégration des équations aux dérivées partielles linéaires à coefficients constants dans les travaux de Cauchy (1821-1830), Études sur Cauchy (1789-1857), Rev. Histoire Sci. 45 (1) (1992), 83-114.
12. Dubbey J. M.: Cauchy's contribution to the establishment of the calculus, Ann. of Sci. 22 (1966), 61-67.
13. Fisher G.: Cauchy and the infinitely small, Historia Mathematica 5 (1978), 313-331.
14. Fisher G.: Cauchy's variables and orders of the infinitely small, British J. Philos. Sci. 30 (3) (1979), 261-265.
15. Freudenthal, H. Did Cauchy plagiarise Bolzano, Archive for History of Exact Sciences 7 (1971), 375-392.
16. Gario P.: Cauchy's theorem on the rigidity of convex polyhedra (Italian), Archimede 33 (1-2) (1981), 53-69.
17. Gilain C.: Cauchy et le cours d'analyse de l'École Polytechnique, Bull. Soc. Amis Bibl. École Polytech. (5) (1989), 1-145.
18. Grabiner J. V.: Who gave you the epsilon? Cauchy and the origins of rigorous calculus, Amer. Math. Monthly 90 (3) (1983), 185-194.
19. Grabiner J. V.:, The origins of Cauchy's theory of the derivative, Historia Math. 5 (4) (1978), 379-409.
20. Grattan-Guinnes I.: Bolzano, Cauchy and the 'new analysis' of the early nineteenth century, Archive for History of Exact Sciences 6 (1970), 372-400.
21. Grattan-Guinness I.: The Cauchy - Stokes - Seidel story on uniform convergence again: was there a fourth man?, Bull. Soc. Math. Belg. Sér. A 38 (1986), 225-235.
22. Grattan-Guinness I.: On the publication of the last volume of the works of Augustin Cauchy, Janus 62 (1-3) (1975), 179-191.
23. Gray J.:, Cauchy - elliptic and abelian integrals, Études sur Cauchy (1789-1857), Rev. Histoire Sci. 45 (1) (1992), 69-81.
24. Gui Z.L.- Zhao D. F., Cauchy: an outstanding mathematician (Chinese), J. Central China Normal Univ. Natur. Sci. 23 (4) (1989), 597-60
25. Hawkins T.: Cauchy and the spectral theory of matrices, Historia Mathematica 2 (1975), 1-29.
26. Koetsier T.: Cauchy's rigorous calculus: a revolution in Kuhn's sense?, Nieuw Arch. Wisk. (4) 5 (3) (1987), 335-354
27. Smithies F.: The background to Cauchy's definition of the integral, Math. Centre Tracts 149 (Amsterdam, 1982), 93-100.
28. Struik D.- Struik R.: Cauchy and Bolzano in Prague, Isis 11 (1928), 364-366.
29. Taton R.: Sur les relations scientifiques d'Augustin Cauchy et d'Évariste Galois, Rev. Histoire Sci. Appl. 24 (2) (1971), 123-148.
30. Terracini A.: Cauchy a Torino, Univ. e Politec. Torino. Rend. Sem. Mat. 16 (1956/1957), 159-203.