**International Journal of Advanced Engineering and Management Research**

Vol. 6, No. 03; 2021

ISSN: 2456-3676

# THE RIGOR IN ANALYSIS MATHEMATICS

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**Abstract**
Cauchy was a mathematician who gave pure mathematics a great and important impetus; for fecundity and variety of production, he can be compared only to Euler: his writings, published during forty-seven years of continuous work, in separate volumes or in scientific collections, are about 789. To prevent this immense work from being lost, the Académie des Sciences in Paris began publishing the "Oeuvres complétes" as early as 1882, which is not yet finished. In this paper, we will examine rigor in analysis mathematics.

**Keywords:** The Rigor in Analysis Mathematics, Mathematics, History

## 1. The Rigor in Analysis Mathematics

Modern mathematics owes Cauchy two very interesting innovations, each of which marks a sudden turning point in the path followed by eighteenth-century mathematics. The first is the introduction of rigor in mathematical analysis. Before that, analysis was a pantheon of false gods, which Cauchy, along with Gauss[1] and Abel[2], completely overthrew, but it was Cauchy who worked as a great pioneer in this field. Gauss could have been at the head of the movement before Cauchy entered the arena, but he did not, so only Cauchy's speed in publishing his studies and his teaching skills imposed rigor in mathematical analysis for the first time. In fact, the imprecision of the concepts and proofs of vast branches of analysis have long worried mathematicians. The concept of function itself was not clear; the use of series with regard to convergence and divergence had produced paradoxes and dissensions; the controversy over the representation of functions by means of trigonometric series had introduced further confusion; and, of course, the fundamental concepts of derivative and integral had never been correctly defined. All these difficulties ended up generating profound dissatisfaction with the logical state of the analysis. Numerous mathematicians decided to bring order to this chaos. The rigorous analysis began with the work of Bolzano[3], Cauchy, Abel and Dirichlet[4] and was then pursued by

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[1] Johann Friedrich Carl Gauss (1777 -1855 ) was a mathematician, astronomer and physicist German, who gave contributions determinants mathematical analysis, number theory, statistics, numerical calculation, differential geometry, geodesy, geophysics, magnetism, electrostatics, astronomy and optics. Sometimes referred to as "the Prince of mathematicians" as Euler or "the greatest mathematician of modernity" (as opposed to Archimedes, considered by Gauss himself as the greatest mathematician of antiquity), he is counted among the most important mathematicians in history having contributed decisively to the evolution of the mathematical, physical and natural sciences. He defined mathematics as "the queen of the sciences".

[2] Niels Henrik Abel (1802 -1829 ) was a Norwegian mathematician, best known for his fundamental contributions to algebra and the theory of functions. Abel's name received major recognition. The institution of the "Abel Prize", established to fill the gap of the Nobel who continues to ignore mathematics.-

[3] Bernard Placidus Johann Nepomuk Bolzano (1781 - 1848 ) was a Bohemian mathematician, philosopher, theologian, presbyter and logician who wrote in the German language making significant contributions to both mathematics and the theory of knowledge.

[4] Johann Peter Gustav Lejeune Dirichlet, (1805 - 1859 ), was a German mathematician, best remembered for the modern "formal" definition of function.

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Weierstrass[5]; but only with Cauchy did "rigorous in analysis" methods finally prevail. In reality, even that of rigor is a historically determined concept: thus, for example, the algebraic methods of Lagrange[6], which Cauchy opposed, were certainly not considered not very "rigorous" by the author of the theory of derived functions nor by mathematicians such as Poisson[7] or Ampère[8], on the contrary; even the "geometric rigor" of which Cauchy will boast so much in his studies will in turn be accused of being scarcely "rigorous" by men like Darboux[9], Dedekind[10] or Weierstrauss. All this seems to suggest a reading of the development of mathematics, and of analysis in particular, dominated by a continuous search for ever greater rigor. In fact, the image of mathematicians tormented by fundamental questions, by a perennially unsatisfied anxiety of rigor and by the nightmare of counterexamples, will perhaps belong to the "rational reconstructions" of the history of mathematics, but it has little to do with real dynamics historical. In reality, these "reconstructions", privileging the "foundational" aspects, not only end up ignoring a large part of the body of actual mathematical research but, also as regards the foundations, seem to forget that new standards of rigor have historically established themselves after that the old criteria had proved incapable of answering the questions that came from mathematical practice or from problems often of a physico-mathematical nature, which, treated mathematically, required changes in the theoretical framework. Thus, for example, at the beginning of the nineteenth century the discussions on the foundations of calculus, on limits and infinitesimals, were intertwined with those on the principles of mechanics, on the calculus of improper integrals, on the methods of integration of differential equations or on the difficult questions of analyzes raised by Fourier research on heat propagation. At that time Paris represented the center of mathematical research and the mathematicians of other countries

[5] Karl Theodor Wilhelm Weierstrass (1815 -1897) was a German mathematician often cited as the "father of modern analysis". Despite leaving university without a degree, he studied mathematics and trained as a school teacher, eventually teaching mathematics, physics, botany and gymnastics. He later received an honorary doctorate and became professor of mathematics in Berlin. Among many other contributions, Weierstrass formalized the definition of the continuity of a function, proved the intermediate value theorem and the Bolzano–Weierstrass theorem, and used the latter to study the properties of continuous functions on closed bounded intervals.

[6] Joseph-Louis Lagrange, (1736 - 1813), was an Italian mathematician and astronomer active, in his scientific maturity, for twenty-one years in Berlin and twenty-six in Paris. He is unanimously considered one of the greatest and most influential European mathematicians of the eighteenth century. His most important work is the Mécanique analytique, published in 1788, with which rational mechanics was conventionally born. In mathematics, he is remembered for his contributions to number theory, for being one of the founders of the calculus of variations, deducing it in the "Mecánique", for having outlined the foundations of rational mechanics, in the formulation known as Lagrangian mechanics, for the results in the field of differential equations and for being one of the pioneers of group theory. In the field of celestial mechanics he conducted research on the phenomenon of lunar libration and, later, on the movements of the satellites of the planet Jupiter; he investigated the problem of the three bodies and their dynamic equilibrium with the rigor of mathematical calculation. His pupils were Jean-Baptiste Joseph Fourier, Giovanni Plana and Siméon-Denis Poisson.

[7] Siméon Denis Poisson (1781 - 1840 ) was a mathematician, physicist, astronomer and Statistical French. Of modest origins, he was encouraged to study and entered the *École Polytechnique* in Paris in 1798. He became a teacher at this school also thanks to the support of Laplace and in 1806 he succeeded Fourier. In 1816 he obtained a chair of mechanics at the Sorbonne and was elected to the Paris Academy of Sciences.

[8] André-Marie Ampère ( Lyon , January 20, 1775 - Marseille , June 10, 1836 ) was a French physicist . He made fundamental contributions in the study of electric current, developing the first mathematical models for the description of the phenomena of electromagnetism. In his honor, the homonymous unit of measurement for electric current is adopted in the International System. His name appears among the 72 names engraved on the Eiffel Tower.

[9] Jean Gaston Darboux ( Nîmes, August 14, 1842 - Paris, February 23, 1917 ) was a French mathematician. He contributed significantly to geometry and mathematical analysis, such as in the field of differential equations. He was the biographer of Henri Poincaré. He received his degree from the École norma supérieure in 1866. In 1884 he was elected to the French Academy of Sciences. In 1900 he was appointed permanent president of the Academy.

[10] Julius Wilhelm Richard Dedekind ( Braunschweig , October 6, 1831 - Braunschweig , February 12, 1916 ) was a German mathematician . He made important contributions to number theory, working closely with Ernst Eduard Kummer.

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looked to the French capital. Looking at the events that animated Parisian scientific life from this perspective, we certainly lose in richness, in details, but we can see more clearly what differences in conceptions and methods then separated the proponents of the Lagrangian theory of functions derived from modern analysis, of which Cauchy was the spokesman for. Cauchy in the course of his studies aimed at seeking rigor, was taken by some scruples for the ostracism shown to the divergent series. Cauchy's main works on the foundations of analysis are the Cours d'analyse algébrique (1821), the Résumé des lecons sur le calcul infinitésimal (1823) and the Lecons sur le calcul différential (1829). Cauchy stated very explicitly in the Cours d'analyse algébrique of 1821 that his goal was to make the analysis rigorous, noting, among other things, that the free use for all functions of the properties that hold for algebraic functions and use of divergent series are not justified. Although his work was only the first step in the direction of rigor, Cauchy was convinced, and affirms it in the Résumé, that he had introduced definitive rigor into the analysis. Solicited by men like Laplace and Poisson and "for the greater benefit of the students", Cauchy in 1821 published the first part of his lecture course, the analyse algébrique, which was aimed primarily at first-year students of the École Polytecnique. Animated by a conception of analysis and rigor not unlike that of Bolzano[11], however, he managed to ensure that his work found much more acceptance in the mathematical world than the Bolzano "pamphlet", thus quickly becoming the manifesto of " new analysis ", it was a volume" which must be read by any analyst who loves rigor in mathematical research, " wrote Abel. From the introductory pages to the Cours d'analyse Cauchy expressed with great vigor the concepts that had guided him in establishing the foundations of mathematical analysis: "As for the methods, I tried to give them all the rigor that is required in geometry in order to never resort to arguments taken from the generality of algebra. Arguments of this type, although very commonly admitted especially in the transition from converging series to those divergent and from real quantities to imaginary expressions, it seems to me that they can only be considered as inductions suitable for sometimes making the truth present, but which do not agree with the much vaunted accuracy of the mathematical sciences. It should also be noted that they tend to attribute an indefinite extension to algebraic formulas, while in reality most of these formulas exist only under certain conditions and for certain values of the quantities contained therein. By determining these conditions and these values and by precisely fixing the meaning of the notations I use, I make all uncertainty disappear… ... It is true that, in order to keep myself constantly faithful to these principles, I was forced to admit various propositions that may seem a bit harsh at first sight… ". The first of the "somewhat hard" propositions to be admitted is the fact that "a divergent series has no sum"; something that must have been very difficult in the eyes of contemporaries, if Cauchy highlights it with such prominence. And in fact it was clearly opposed to a tradition that had always been dominant in analysis and still reaffirmed in Lagrange's Traité des fonctions analytiques, also one of the books most studied by the young Cauchy. But if he agrees with Lagrange on the need to found the analysis rigorously, without limiting himself to justifying the methods with success in the applications to geometry, physics, etc., he nevertheless distances himself clearly from him when it comes to identifying the

[11] Bernard Placidus Johann Nepomuk Bolzano (1781 - 1848 ) was a Bohemian mathematician, philosopher, theologian, presbyter and logician who wrote in the German language making significant contributions to both mathematics and the theory of knowledge.

foundations: arguments drawn from algebra, says Cauchy, opposing Lagrange, cannot serve as a basis for the "much vaunted accuracy" of the analysis The infinite series play a decisive role in these questions, and the rigor in dealing with them must be extreme; even at the cost of drastic reductions in the extent of the formulas used. “So, before carrying out the sum of any series, I had to examine in which cases the series can be added, or, in other words, what are the conditions for their convergence; and, in this regard, I have established some general rules which seem to me to deserve some attention ”. The tool that Cauchy develops to complete his critical revision is the theory of limits; it is the concept of limit that allows him to define the continuity of functions, the derivative and the integral, the convergence of a series and its sum. The Cours d'analyse, as is natural for a general and didactically effective treatment, opens with a series of preliminaries, where Cauchy reviews the different kinds of numbers (natural, relative, etc.), introduces the concept of absolute value, which he calls "numerical value", the calculation with literal quantities and finally the concept of limit, which he defines as follows: "When the values subsequently assumed by the same variable indefinitely approach a fixed value, so as to differ in the end so little as much as you want, this last quantity is called the limit of all the others ". It is well known how important it was for mathematics to have isolated this concept more or less clearly present in the spirit of every mathematician, but never before placed at the foundation of infinitesimal calculus. It is interesting to note the example that Cauchy immediately presents to illustrate the concept: "So for example, an irrational number is the limit of the different fractions that provide ever more approximate values". The introduction of limits allows Cauchy to unequivocally specify the meaning of infinitesimal and positive and negative infinity. Cauchy states “as successive numerical values of the same variable decrease indefinitely to become less than any given number, this variable becomes what is called an infinitesimal or an infinitesimal quantity. When the successive numerical values of the same variable grow more and more, so as to exceed any given number, it is said that this variable has the limit of positive infinity, indicated with $+\infty$if it is a positive variable; instead it is said that this variable has as limit the negative infinity indicated with $-\infty$, if it is a negative variable ". Finally Cauchy presents the usual operations of calculus, sum, product, etc., the exponential and the logarithm and the trigonometric functions. In the first chapter you will immediately find the definition of a function of one and more real variables. "When variable quantities are linked together in such a way that, given the value of one, we can obtain the value of all the others, expressed by means of the independent variable, they are called functions of this variable". In a similar way he defines the functions of several independent variables and distinguishes between explicitly defined functions and implicit functions, a case that occurs when only the relations between the functions and the variable are given, i.e. the equations to which these quantities satisfy, without that such equations are solved algebraically. After defining the infinitesimals of first order and successive orders through the limits, Cauchy gives the following definition of continuity of a function: "Let $f(x)$ be a function of the variable $x$, and suppose that for any intermediate value of $x$within two given limits, the function always admits a finite value ". If, starting from a value of $x$ included within these two limits, is attributed to the variable$x$, an infinitesimal increment α, the function itself will receive the difference as an increment:

$$f(x+\alpha)-f(x)$$

which will depend at the same time on the new variable α, and on the value of $x$.

Having said that, the function $f(x)$ will be, within the two limits assigned to the variable $x$, a continuous function of this variable if, for each value of $x$ between these two limits, the numerical value of the difference:

$$f(x+\alpha)-f(x)$$

it will decrease indefinitely together with that of α. At this point he follows the reformulation in terms of infinitesimals of the same concept:

*The function $f(x)$ will remain continuous with respect to $x$ between the two given limits, if, within these limits, an infinitesimal increment of the variable always produces an infinitesimal increment of the function itself.*

The definition of function that Cauchy gives appears completely free from the expressibility, through "an analytic expression", of the dependent variable (as it was for Lagrange, for example) as well as continuity, understood by Cauchy, also includes functions with angular points and therefore not everywhere derivable in their domain of definition; as in Bolzano, Cauchy also clearly expresses the idea that the function should be considered "within two given limits" when, for example, one wishes to affirm something about its continuity.

Bolzano's and Cauchy's definitions of a continuous function appear strikingly similar, which seems even more remarkable when one considers that at the time this was an entirely new way of studying continuity. But, while Bolzano seems consciously towards the distinction between continuity and derivability[12] Cauchy seems to be here still linked to classical analysis, and produces standard examples of continuous functions:

$$(a+x, a-x, ax, \sin x, \cos x, \log x, A^x\ etc)$$

All differentiable and as a discontinuous function gives the example of $a/x$ for $x=0$.

Two years later, publishing the Resumé of the lectures given at the École polytechnique, Cauchy wrote that "*the two function $x^{\frac{1}{2}}$ and $\frac{l}{\log x}$ become discontinuous passing from the real to the imaginary when the variable $x$ decreases passing through zero*".

The sense of this statement seems to be that Cauchy thinks that continuous functions are always differentiable, and cease to be only at points of discontinuity. In fact, let's take for example the first of the functions proposed by Cauchy:

[12] In the Funktionenlebre (unpublished) of 1830, Bolzano considers a continuous function in every point but in no derivable point, of which the first published example appears only in the seventies and, in the Concepts of Weierstrass, a little earlier

$$y = x^{\frac{1}{2}}$$

whose derivative is:

$$y' = \frac{1}{2\sqrt{x}}$$

Which represents the discontinuity of the origin. If we move from real to complex values, it becomes clear immediately, says Cauchy. In this case, in fact, the point $x = 0$turns out to be a multiple point, and the function changes its determination when the variable makes a complete revolution around it. Several years later Cauchy himself had occasion to clarify this idea better. Explaining in a letter to Coriolis[13], published in the accounts of the Academy of Sciences (1837), his own method, based on convergent series developments, to represent the roots of algebraic equations or the integrals of differential equations, he wrote about continuity: "According to the definition given in my Cours d'analyse, a function of a variable t continues within given limits, when within these limits each value of the variable gives rise to a unique and finite value of the function and when it varies by insensitive degrees with the variable itself. That said, a function that does not become infinite does not generally cease to be continuous except by becoming multiple ". A full awareness of the theoretical novelty, inherent in his own definition of continuity, with respect to the Eulerian and Lagrangian radiation, is found, in Cauchy, only in a much later writing, in which we read: " In the works of Euler[14] and Lagrange, a function is called continuous or discontinuous, according to the different values of it, corresponding to

[13] Gaspard-Gustave de Coriolis (1792 - 1843 ) was a mathematician, physicist and mechanical engineer French. In 1816 Coriolis became an assistant at the École Polytechnique where he carried out experiments on friction and hydraulics . His career overlapped with the start of the industrial revolution, cntered on using the steam engine with its rapidly rotating mechanical systems. His interest in the dynamics of rotating machines led him to formulate the differential equations of motion from the point of view of a coordinate system in turn in rotation, presented to the French Academy of Sciences in 1831. Its name is linked to the Coriolis force, a mechanical force predicted by his theorem, which has become of great importance in meteorology to explain the formation of atmospheric vortices. An asteroid was dedicated to him, 16564 Coriolis [1] and a lunar crater, Coriolis Crater.

[14] Leonhard Euler (1707 - 1783 ) was a Swiss mathematician and physicist. He is considered the most important mathematician of the eighteenth century, and one of the greatest in history. It is known to be among the most prolific of all time and has provided historically crucial contributions in several areas: calculus, special functions, rational mechanics, celestial mechanics, number theory, graph theory. Pierre Simon Laplace seems to have said "Read Euler; he is the teacher of us all". Euler was undoubtedly the greatest supplier of "mathematical denominations", offering his name to an impressive amount of formulas, theorems, methods, criteria, relations, equations. In geometry: the *circle*, the *line* and *Euler's points* relative to the triangles, plus the *Euler-Slim relation*, which concerned the circle circumscribed to a triangle; in number theory: *Euler's criterion* and Fermat-Euler's theorem, Euler's *indicator, Euler*'s *identity, Euler*'s *conjecture*; in mechanics: the *Euler angles*, the *critical load Euler*(due to instability); in the analysis: the *Euler-Mascheroni constan* , the Euler gamma function; in logic: the *Euler-Venn diagram*; in graph theory: (again) the *Euler relation*; in algebra: *Euler's method* (relating to the solution of the fourth degree equations), Euler's Theorem; in differential calculus: *Euler's method* (concerning differential equations). Other mathematical objects are also linked to Euler, through the adjective "Eulerian", such as: the *Eulerian cycle*, the *Eulerian graph*, the *Eulerian function of the first kind* or *beta function*, and that of the second kind or *gamma function*, the *Eulerian chain of a graph without loops*, the *Eulerian numbers* (different from Euler's Numbers ). Although he was predominantly a mathematician he made important contributions to physics and in particular to classical and celestial mechanics. For example, he developed the Euler-Bernoulli equation of beams and the Euler-Lagrange equations. It also determined the orbits of many comets. Euler kept in touch with numerous mathematicians of his time; in particular he had a long correspondence with Christian Goldbach comparing some of his own results with him. He also knew how to coordinate the work of other mathematicians who were close to him: his sons Johann Albrecht Euler and Christoph Euler, the members of the St. Petersburg Academy WL Krafft and Anders Johan Lexell and his secretary Nicolaus Fuss (who was also the husband of her granddaughter); to all the collaborators he recognized the merits. Altogether there are 886 publications of Euler. Much of the mathematical symbology still in use was introduced by Euler, for example, $i$ for the imaginary unit, $\Sigma$ as a symbol for summation, $f(x)$ to indicate a function and the letter $\pi$ to indicate pi.

different values of the variable, are or are not subject to the same law, are or are not provided by one and the same equation ". It is in those terms that the continuity of functions is defined by these illustrious geometers, when they said that arbitrary functions, introduced by the integration of partial differential equations, can be continuous or discontinuous functions. However, such a definition is far from offering mathematical precision, since, if the different values of a corresponding function depend on two or more distinct equations, nothing will prevent us from decreasing the number of these equations, and even from replacing them with a single equation. whose decomposition would provide all the others. More: the analytic laws, to which functions can be subjected, are generally found expressed by algebraic or transcendent formulas, and it may happen that different formulas represent, for certain values of a variable$x$, the same function, and for other values of $x$, of different functions. Therefore, if we consider Euler's and Lagrange's definitions as applicable to every kind of function, whether algebraic or transcendent, a simple change of notation will often suffice to transform a continuous function into a discontinuous function, and reciprocally. In the same chapter of the Cours d'analyse, after defining the continuity of compound functions, Cauchy dedicates long pages to the study of the "singular values of functions in some particular cases. It is "one of the most important and most delicate questions of analysis", says Cauchy, that is to study the limits of some functions for

$$\infty^+, \qquad \infty^-, \qquad x = 0$$

which leads him to identify the so-called "indeterminate forms" of the type:

$$\frac{0}{0}, \quad \frac{\infty}{\infty}, \quad 0 * \infty, \quad 1^\infty, \quad \infty^0$$

another very important point object of study in the Cours d'analyse was the study of the infinite series. An infinite series was called convergent by Cauchy if the sum sn of the first $n$ terms of the series "approaches indefinitely to a certain limit s" for ever increasing values of$n$. On the contrary, if as $n$increases$s_n$ does not approach any fixed limit, the series will be divergent, and will no longer have a sum". Cauchy demonstrated in this course the necessary and sufficient condition that, for infinitely large values of the number n, the sums$s_n$ $s_{n+1}$ $s_{n+2}$, differ from the limit$s$, and consequently between them, by infinitesimal quantities because the sum $s_n$converges indefinitely towards a fixed limit$s$. Among the immediate applications of his criterion, Cauchy stated an apparently very convincing theorem: given a series of functions of the same variable x, all continuous around a certain particular value of the variable for which the series is convergent, also the sum $s$of the series will be a continuous function of$x$ around that particular value. Therefore, in the light of the foregoing, we can certainly affirm that only with Cauchy did the "rigorous" methods in analysis prevail.

**2. Conclusions**

In light of the above, we can certainly affirm that only with Cauchy did the "rigorous" methods i analysis prevail. Today, this elementary theory, although complicated, is of capital importance in many fields of pure and applied mathematics, from the theory of algebraic equations to geometry and the theory of atomic structure; to mention only one of its applications, we find it at the basis of crystal geometry, its further developments, from the analytical side, extend far beyond in higher mechanics and in the modern theory of differential equations. In light of the foregoing, we can highlight that one of the often overlooked elements is the fact that, as in many other sciences, also in mathematics, new theories and new points of view have historically established themselves through discussions, polemics and sometimes decided contrasts concepts and methods.

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