International Journal of Advanced Engineering and Management Research

Vol. 6, No. 04; 2021

ISSN: 2456-3676

MATHEMATICS: INNOVATION AND PROGRESS

Boniello Carmine
University of Salerno Fisciano (Italy)

Abstract
Mathematics is innovation and progress. The most original creation of the 19th century was the theory of functions of a complex variable which was also hailed as one of the most harmonious theories of all abstract sciences. This discipline is often referred to as function theory, although the abbreviated description implies more than expected. This new branch of mathematics dominates the whole of the 19th century. Starting from this awareness, this article has been structured by examining the contributions that Cuchy has made to the theory of complex variables but also to closed solutions and the Fourier integral. Finally, we will examine his paper Il memoire sur les function.

Keywords: The theory of complex variables, function, the Fourier integral

1. The theory of complex variables

Technically, the most original creation of the 19th century was the theory of functions of a complex variable which has been hailed as one of the most harmonious theories of all abstract sciences. This discipline is often cited as a theory of functions, although the abbreviated description implies more than is intended. This new branch of mathematics dominates the whole of the 19th century. Cauchy's first significant work in the direction of the theory of complex functions is the Mémoire des intégral définies, read in front of the Paris Academy in 1814. In the publication Cauchy[1] added two notes that almost certainly develop developments that occurred between 1814 and 1825 and the possible influence of Gauss[2]'s work during that period. Cauchy says in the preface that he was induced to this research by the attempt to rigorously the passage from the real to the imaginary in the procedures used by Euler[3] since 1759 and by Laplace[4] since

[1] Augustin-Louis Cauchy (1789 -1857) was a mathematician and French engineer. He started the project of the rigorous formulation and demonstration of the theorems of the infinitesimal analysis based on the use of the notions of limit and continuity. He also gave important contributions to the theory of functions of complex variable and the theory of differential equations. The systematicity and the level of these works places it among the fathers of the mathematical analysis. Cauchy's genius is evident in his simple solution to Apollonius 's problem , namely the description of a circle touching three other data circles he discovered in 1805 , the generalization of Euler 's characteristic for polyhedra in 1811 , and many other problems solved elegantly. Of great importance are his writings on wave propagation, thanks to which he obtained the Institute's Grand Prix in 1816 . His greatest contributions to mathematics are enshrined in the rigorous methods he introduced. This is found mainly in his three major treatises: Cours d'analyse de l ' École Polytechnique (1821); Le Calcul infinitésimal (1823); Leçons sur les applications de calcul infinitésimal ; La géométrie (1826 - 1828); and also in his Courses of mechanics (for the École Polytechnique), Higher algebra (for the Faculté des Sciences), and of Mathematical physics (for the Collège de France).

[2] Johann Friedrich Carl Gauss (1777 -1855) was a mathematician , astronomer and physicist German , who gave contributions determinants mathematical analysis , number theory, statistics , numerical calculation, differential geometry, geodesy , geophysic, magnetism , electrostatics , astronomy and optics.

[3] Leonhard Euler, (1707 - 1783), was a Swiss mathematician and physicist .He is considered the most important mathematician of the XVIII century , and one of the greatest in history. It is known to be among the most prolific of all time and has provided historically crucial contributions in several areas: calculus , special functions , rational mechanics , celestial mechanics , number theory , graph theory . Pierre Simon Laplace seems to have said "Read Euler; he is the teacher of us all".

[4] Pierre-Simon Laplace , Marquis de Laplace (1749 - 1827), was a mathematician , physicist , astronomer and noble French . He was one of the leading scientists of the Napoleonic period , appointed Minister of the Interior in 1799 by Napoleon , who in 1806 conferred on him the title of

1782 to underestimate the definite integrals, and in fact Cauchy quotes Laplace, who had observed that the method needed to be made rigorous. However, the work itself does not deal with this problem, but rather the inversion of the order of integration in the double integrals that arise from research on thermo hydraulics. Euler had said in 1770 that this inversion is admissible when the limits of each of the variables appearing under the integral sign are independent of each other and Laplace agreed because he uses this due fact. Specifically Cauchy considers the relationship:

$$\int_{x_0}^{x}\int_{y_0}^{y} f(x,y)\,dydx = \int_{y_0}^{y}\int_{x_0}^{x} f(x,y)\,dxdy$$

Where x_0 and y_0, X and Y are constant. This inversion of the order of integration is valid when $f(x,y)$ it is continuous in the region and on the border. It then introduces two functions $V(x,y)$and $S(x,y)$ such that:

(1) $$\frac{\partial v}{\partial y} = \frac{\partial s}{\partial x} \qquad \frac{\partial v}{\partial y} = \frac{-\partial s}{\partial x}$$

Euler demonstrated in 1777 how such functions had already been derived. Cauchy then considers:

$$f(x,y) = \frac{\partial s}{\partial x}$$

which is given by:

$$\frac{\partial v}{\partial y} = \frac{\partial s}{\partial x}$$

and replaces in the initial relation $f(x,y)$ to the first member with

$$\frac{\partial v}{\partial y}$$

and to the second member with:

$$\frac{\partial s}{\partial x}$$

so we have:

Count of the Empire , then also appointed Marquis in 1817, after the restoration of the Bourbons . He made fundamental contributions in various fields of mathematics, astronomy and probability theory and was one of the most influential scientists of his time, also for his contribution to the affirmation of determinism . He put the final twist on mathematical astronomy, summarizing and extending the work of his predecessors in his five-volume work Mécanique Céleste (Celestial Mechanics) (1799 - 1825). This masterpiece transformed the geometric study of mechanics developed by Newton into one based on mathematical analysis.

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ISSN: 2456-3676

$$\int_{x_0}^{x}\int_{y_0}^{y}\frac{\partial v}{\partial y}dydx=\int_{y_0}^{y}\int_{x_0}^{x}\frac{\partial s}{\partial v}dxdy \qquad (2)$$

Using the (1) we get:

$$\int_{x_0}^{x}\int_{y_0}^{y}\frac{\partial s}{\partial y}dydx=-\int_{y_0}^{y}\int_{x_0}^{x}\frac{\partial v}{\partial x}dxdy \qquad (3)$$

These equalities can be used to evaluate double integrals in both orders of integration, but they do not concern complex functions. Cauchy states that these two equations contain the whole theory of the transition from the real to the imaginary. Everything that proceeds is contained in the work of 1814 and there is actually no indication of how this has to do with the theory of complex functions. Furthermore, although Cauchy used complex functions to evaluate definite real integrals in the same way as Euler and Laplace, this use did not involve complex functions as basic entities. Still in 1821, in the Cours d'Analyse he says that:

$$\cos(a)+\sqrt{-I}\sin(a)$$
$$\cos(b)+\sqrt{-I}\sin(b)$$
$$\cos(a+b)+\sqrt{-I}\sin(a+b)$$

"these are three symbolic expressions which cannot be interpreted according to generally established conventions and which do not represent anything real". The fact that the product of the first two previous expressions is equal to the third, he says, makes no sense. To make sense of this equation, you need to equal the real parts and the coefficients of $+\sqrt{I}$. Cauchy also reiterated that: "every imaginary equation is only the symbolic representation of two equations between real quantities". If we operate on complex expressions according to the rules established for real quantities, we obtain exact results which are often important and fundamental. In 1822 Cauchy took another step forward, from relations (2) and (3) we deduced:

$$\int_{x_0}^{x}[V(x;Y)-V(x,y_0)]\,dx=\int_{y_0}^{y}[S(X,y)-S(x_0,y)]dy \qquad (4)$$

$$\int_{x_0}^{x}[S(x;Y)-S(x,y_0)]\,dx=-\int_{y_0}^{y}[V(X,y)-V(x_0,y)]dy \qquad (5)$$

He then had the idea of combining these two equations in order to obtain this statement:

$$F(z)=F(x+iy)=S+IV$$

Multiplying the (4) for i and adding the two equations we obtain:

$$\int_{x_0}^{x} F(x+iY)\,dx - \int_{x_0}^{x} F(x+iy_0)\,dx = \int_{y_0}^{y} F(x+iY)i\,dy - \int_{y_0}^{y} F(x_0+iy)i\,dy$$

rearranging the terms of this equality we obtain:

$$\int_{y_0}^{y} F(x_0+iy)i\,dy + \int_{x_0}^{x} F(x+iY)\,dx = \int_{x_0}^{x} F(x+iy_0)\,dx \int_{y_0}^{y} F(x+iY)i\,dy$$

the latter result is Cauchy's integral theorem in the simple case of complex integration along the boundary of a rectangle. Cauchy expounded these ideas in a note of 1882 to the Résumé des lecons sur le calcul infinitésimal and in a footnote to the work of 1814 which was published in 1827. From these last writings it is inferred that Cauchy passed from real to real functions. complex. In 1825 Cauchy wrote another work entitled "Mèmoire sur les intégrales définies prises entre des limites imaginaires", which was only published posthumously in 1874. This work is considered by many to be his most important and one of the most beautiful in the history of science, even if for a long time Cauchy himself did not appreciate the value. In this paper he again tackles the problem of evaluating integrals with the method of substituting complex values for constants and variables. He consider:

$$\int_{x_0+y_0}^{x+iY} f(z)\,dz \tag{6}$$

where $z = x + iy$ and accurately defines the integral as the limit of the sum:

$$\sum_{v=0}^{n-1} f(xv+iyv)\{[x(v+1)-xv]+i[y(v+1)-y]\}$$

where $x_0, x_1, \dots\dots X$ and $y_0, y_1, \dots\dots Y$ are the subdivision points along a path that goes from (x_0, y_0) to (X, Y). Here $x + iy$ it is undoubtedly a point of the complex plane and the integral is taken along a complex path. Cauchy also demonstrates that if we ask $x = \varphi(t)$ and $\omega(t)$ where t is real, then the result is independent of the choice of φ and ω that is, it is independent of the path, provided that there is no discontinuity between the two different paths. Cauchy formulates his theorem as follows if $f(x + iy)$ is over and continues for:

$$x_0 \leq x \leq X \qquad \text{and} \qquad y_0 \leq y \leq Y$$

then the value of the integral (6) is independent of the form of the functions we $x = \varphi(t)$ and $\emptyset(t)$. The proof of this theorem uses the method of calculating the variations: he considers it as an alternative path $\varphi(t) + \varepsilon_u(t) + \omega(t) + \varepsilon_v(t)$ and shows that the first variation of the integral with respect to ε you cancel. This demonstration is not satisfactory. In it, not only Cauchy uses the existence of the derivative of $f(z)$ but also of its continuity, even if it makes neither of these

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two hypotheses in the statement of the theorem. The explanation lies in the fact that Cauchy believed that a continuous function was always differentiable and that the derivative could only be discontinuous when the function itself was. This opinion of Cauchy was reasonable in that in the first years of his research a function meant for him, as for other mathematicians of the eighteenth and early nineteenth centuries, an analytical expression and the derivative of these expressions is immediately given by the usual formal rules of derivation. In the years ranging from 1830 to 1838, during which he lived in Turin and Prague, his publications were sparodic. Cauchy refers to them by reporting most of the results contained in the Exercises d'analyse et de physique mathématique (1840–1847). In a work of 1831 published later, he demonstrates the following result: the function $f(x)$ can be developed according to Maclaurin's formula in a power series that is convergent for all z whose absolute value is less than those for which the function or its derivative cease to be finite or continuous (the only singularities that Cauchy knew at that time were what we call poles). He showed that this series is a smaller term to term than a convergent geometric series whose sum is equal to:

$$\frac{Z}{Z-z}\overline{f(z)}$$

where Z is the first value for which $f\ (z)$ is discontinuous and $f\ (z)$ is the maximum value of $|f\ (z)|$ for all z whose absolute value is equal to $|Z|$. In this way Cauchy gives a powerful and easy-to-apply criterion for the developability of a Maclaurin series function that uses a reference series now called majorant. In the proof of the theorem he first proves that:

$$F(z) = \frac{I}{2\pi}\int_{-\pi}^{\pi}\left(\frac{\bar{z}\, f(\bar{z})}{\bar{z}-z}\right)d\varphi$$

Where $\bar{z} = |Z|e^{t\varphi}$. This result is basically what we call the Cauchy integral formula. Then develop the fraction $\frac{\bar{z}}{(\bar{z}-z)}$ in a geometric series of powers of $\frac{z}{\bar{z}}$ and proves the theorem proper. Also in this theorem Cauchy assumes that the existence and continuity of the derivative necessarily follow from the continuity of the function. When he reproduced this material in the Exercises he had meanwhile corresponded with Sturm[5] and Liouville[6] on the subject and

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[5] Jacques Charles François Sturm (1803 - 1855) was a French mathematician . In 1821 he began to attend the Geneva Academy where he had Simon Lhuilier as a mathematics teacher who immediately recognized his abilities. After completing his studies at the Academy, in 1823 he became tutor of Madame de Staël 's son and in this capacity he was able to write some articles on geometry that were published in the Annales de mathématiques pures et appliquées directed by Joseph Diaz Gergonne . At the end of 1823 Madame de Staël's family moved to Paris, Sturm followed her and was able to participate in scientific meetings organized by François Arago and attended by personalities such as Laplace , Poisson , Fourier , Gay-Lussac and Ampere . Back in Geneva, he then decided, together with his schoolmate Colladon , to seek his fortune in Paris , and obtained a job at the Bulletin universel .

In 1829 he discovered the theorem relating to the determination of the number of real roots of a numerical equation between two given limits, a theorem that bears his name. In the following year he was appointed professor of mathematics at Rollin College . He was admitted as a member of the Académie des Sciences in 1836 , became répétiteur in 1838 , and in 1840 professor at the École Polytechnique , and finally succeeded Poisson in the chair of mechanics in the Faculty of Sciences in Paris. His works, Cours d'analyse de l'École polytechnique (1857 - 1863) and Cours de mécanique de l'École polytechnique (1861), were published after his death in Paris. The theorem of Sturm is a fundamental result to demonstrate the existence of real zeros of functions.

[6] Joseph Liouville (1809 - 1882) was a French mathematician . Son of a soldier who survived the campaigns of Napoleon Bonaparte and settled in Toul in 1814 , he graduated from the *École Polytechnique* . Subsequently he began the *École nationale des ponts et chaussées* , without however obtaining the diploma for health reasons and for having acquired an inclination to an academic career rather than an engineering one. After a few

therefore added to the statement of the preceding theorem the observation that the convergence region ends with the value of z in which the function and its derivative cease to be finite or continuous. However, he was not convinced of the need to add these conditions to the derivative and in subsequent research he eliminated it. In another fundamental work on the theory of complex functions entitled Sur les intégrales qui sétendent à tous les points d'une courbe fermée Cauchy relates the integral of an analytic function:

$f(z) = u + iv$

along a curve that limits a region it concluded with an integral extended to the whole region.

If u and v sono funzioni di x e y we have:

$$\iint \left[\left(\frac{\partial u}{\partial x}\right) - \left(\frac{\partial v}{\partial y}\right)\right] dxdy = \int u\, dy + \int v\ dx$$

$$\iint \left[\left(\frac{\partial u}{\partial y}\right) - \left(\frac{\partial v}{\partial x}\right)\right] dxdy = -\int u\, dx + \int v\ dy$$

where the double integrals are extended to the region and the simple integrals to the curve that limits them. Now, for the Cauchy-Riemanni equations the first members of these eequalities are equal to 0 and the two second members are the integrals that appear in:

$$\int f(z)dz = \int (u + iv)(dx + i\, dy) = \int (u\, dx - v\, dy) + i \int (u\, dy + v\, dx)$$

Therefore:

$$\int f(z)\ dz = 0$$

in this way Cauchy thus obtains a new proof of the fundamental theorem of the independence of the value of an integral from the integration path. In the aforementioned work of 1846 and in the work entitled "Sur les intégrales dans le squelles la function sous le signe change brusquement de valeur" (1846) Cauchy changed his point of view on the complex functions that had guided him in the researches of 1814, 1825 and 1826. Instead of dealing with definite integrals and their evaluation, he turned to the theory of complex functions proper with the aim of laying the foundations for this theory. In this second work of 1846 he gives a new statement concerning the integral $\int f(z)\, dz$ along an arbitrary closed curve: if the curve closes the poles, then the value of the integral is equal to $2\pi i$ times the sum of the residuals of the function in these poles, that is:

$$\int f(z)\ dz = 2\pi i\ E[f(z)]$$

where $E[f(z)]$ is the notation he uses to indicate the sum of the residuals. Cauchy also addressed the issue of integrals of multi-valued functions. In the first part of the work, where he considers

years as an assistant in various institutes, in 1838 he was appointed professor at the École polytechnique. In addition to his mathematical talent, Liouville displayed remarkable organizational skills. He founded the *Journal de Mathématiques Pures et Appliquées* , still highly reputed today. He was the first to read the unpublished works of Évariste Galois , to publish them in his newspaper in 1846 , and to recognize their importance. Liouville was also politically engaged for a short time: a member of the constituent assembly in 1848 , he left it after an electoral defeat in 1849.

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the integrals of functions at one value, he does not say much more than what Gauss observed in the letter to Bessel[7] about:

$$\int \frac{dx}{x}$$

or

$$\int \frac{dx}{(I + x^2)}$$

integrals are actually multi-valued and their values depend on the path of integration. However, Cauchy goes further as he considers multivalued functions under integral sign. In this regard, he says that if the integrand is an expression of the roots of an algebraic or transcendent equation, such as:

$$\int w^3 dz$$

where $w^3 = z$ and if it is integrated along the closed path it is not independent from the starting point and if one proceeds along the path, different values of the integral are obtained. However, if you continue to turn along the path a sufficient number of times so that w returns to its original value, then the integration values are reproduced and the integral is a periodic function of z. The periodicity modules (indicies de périodicité) of the integral are more representable with residuals, as was the case in the case of single-value functions.

2. Subsequent studies

Cauchy's vague ideas about integrals of multivalued functions were still vague. For about twenty-five years, starting in 1821, Cauchy had been developing the theory of complex functions on his own. In 1843 some of his compatriots began to move along certain lines of his work. Among those who worked on Cauchy's work we remember: Laurent[8], Puiseux[9]. Cauchy wrote other works on integrals of multi-valued functions in which he attempted to continue Puiseux's research, but, even though he introduced the notion of branch cuts (lignes d'arret), he still did not have a clear distinction between the two poles. and branch points. This argument of algebraic functions and their integrals would have been put forward by Riemann[10]. In numerous other

[7] Friedrich Wilhelm Bessel (1784 – 1846) Friedrich Wilhelm Bessel (1784 – 1846) was a mathematician, astronomer and surveyor German. The era in which he lived was dominated by the figure of the great German mathematician Gauss, commonly referred to as the prince of mathematicians . Bessel is remembered, above all, because he was the first to use parallax to measure the distance of a star. He died of retroperitoneal fibrosis in Königsberg.

[8] Pierre Alphonse Laurent (1813 - 1854) was a French mathematician known for his discovery of the Laurent series , an expansion of functions in a power series that generalizes Taylor series . His results are contained in a memoir sent in 1843 for the Grand Prix of the French Academy of Sciences , but as his candidacy was deemed late, the article never entered the competition, and his writings were published posthumously.

[9] Puiseux, Victor-Alexandre (1820 - 1883), first (from 1847) "maître de conférences" at the École normal, then prof. of mathematics and astronomy at the Sorbonne, member of the Bureau des longitudes (1868-72), member of the Academy (from 1871). Most of P.'s works concern celestial mechanics, but his name remains linked above all to a memory on algebraic functions (1851), in the address opened in those years by A. Cauchy with his studies on the functions of complex variables.

[10] Georg Friedrich Bernhard Riemann (1826-1866) was a German mathematician and physicist. He contributed significantly to the development of the mathematical sciences. Among his works in the mathematical field are those related to geometry, of which he revolutionized the approach

works that appeared in the “Comptes rendus” of 1851, Cauchy stated with more precision some properties of complex functions. In particular, he stated in them that the continuity of the derivatives, as well as the continuity of the complex function itself, are necessary for the develop ability in a power series. He also observed that the derivative in $z = a$ of u considered as a function of z is independent of the direction of the plan $x + iy$ along which z approaches a is that u satisfy the equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

In these works of 1851 Cauchy introduced new terms. He calls "monotypique" and also "monodrome" a function that is single-valued for each value of z in some domain. A function is called "monogenic" if for each z it has only one derivative (that is, if its derivative is independent of the path). A "monodrome and monogen" function that never becomes infinite is called "synectique". We can certainly conclude by affirming that as far as complex functions are concerned, the nineteenth century ended with a return to the fundamentals, and the French mathematician was one of the greatest founders of the theory of functions. The other founders of the theory of functions were Riemann and Weirstrass[11], but the complete unification of their works only took place at the beginning of the twentieth century

3. The closed solutions and the Fourier integral

Despite the success and impact of Fourier solutions, one of the main efforts during the nineteenth century was the request for solutions of partial differential equations in closed form, that is, in terms of elementary and integral functions of these functions. These solutions, at least those of the type known in the eighteenth and early nineteenth centuries, were more manageable, more perspicuous and easier to use for calculation. The most significant method for solving partial differential equations in closed form, originating from Laplace's research, was the Fourier integral. The idea is due to Fourier[12], Cauchy and Poisson[13]. It is impossible to prioritize this

to the study (Riemann surfaces, Riemann sphere, Riemann tensor), those relating to analysis, even complex (Riemann integral, Function zeta of Riemann) and those on prime numbers, with the relative hypothesis.

[11] Karl Theodor Wilhelm Weierstrass (1815-1897) was a German mathematician, often called the "father of modern analysis". He dealt with rigorously defining the foundations of the analysis, first giving the example of a function that is continuous everywhere but not derivable. Its name is linked to the Weierstrass theorem, the Bolzano-Weierstrass theorem and the Weierstrass criterion for the uniform convergence of series.

[12] Jean Baptiste Joseph Fourier (1768 - 1830) was a French mathematician and physicist . His education was first carried out by the Benedictines , then in a military school. He participated in the French Revolution , risking being guillotined during the Terror , but was saved by the fall of Robespierre . He then entered the École Normale Supérieure , where he had as professors, among others, Joseph-Louis Lagrange and Pierre-Simon Laplace . He also succeeded the latter in the role of professor at the École Polytechnique in 1797 .Fourier participated in the Egyptian campaign of Napoleon in 1798 , and played an important role as a diplomat in that country. Upon his return to France in 1801 , he was appointed prefect of Isère by Napoleon . It was then there, in the city of Grenoble , that he conducted his experiments on the propagation of heat which allowed him to model the evolution of temperature by means of trigonometric series. These works were published in 1822 in *Analytical Theory of Heat* , but were much contested, especially by Laplace and Lagrange . In 1817he joined the Academy of Sciences. In Grenoble he met the young Jean-François Champollion . His major contributions include: the theorization of the Fourier series and the consequent Fourier transform in mathematics, the formulation of the linear constitutive law for thermal conduction and Fourier's law in thermodynamics.

[13] Siméon Denis Poisson (1781 - 1840) was a mathematician , physicist , astronomer and Statistical French . Of modest origins, he was encouraged to study and entered the *École Polytechnique* in Paris in 1798 . He became a teacher at this school also thanks to the support of Laplace and in 1806 he succeeded Fourier . In 1816 he obtained a chair of mechanics at the Sorbonneand was elected to the Paris Academy of Sciences . Among his contributions, he extended the theory of mechanics using analytical mechanics (*Traité de mécanique* , 2 volumes, 1811 and 1833). He then showed that a particle placed between two ellipsoidal plates oriented in the same direction, does not feel any force. He also contributed to the development of statistics with the *Poisson distribution*.

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very important discovery because all three presented orally to the Académie des Sciences some works on how to solve partial differential equations in closed form, but these works were published only after a certain period of their presentation, and therefore historians of mathematics have never been able to establish which of the three had the first place in being the solution to the equations. The only certainty is that each of the three great mathematicians listened to the works of the others and it is impossible to say, on the basis of the publications, what each of them may have taken from these verbal accounts. Cauchy himself made the derivation of the Fourier integral which is quite similar. The work in which it appears entitled "Théorie de la propagation des ondes", received the prize of the Parisian Académie des Sciences in 1816. It constitutes the first extensive research on the superficial waves of a fluid, a topic that had been addressed for the first time in 1816 by Laplace. Although he writes the hydrodynamic equations, Cauchy limits himself almost from the beginning to considering special cases. In particular, he considers the equation:

$$\frac{\partial^2 q}{\partial x^2}+\frac{\partial^2 q}{\partial y^2}=0$$

in which q is what was later called potential speed and x and y are spatial coordinates, and he writes the solution, without giving explanations:

$$q=\int_0^{\infty}\cos(mx)\,e^{-ym}f(x)dm$$

where $f(m)$ is finished at this arbitrary moment. Since y equals 0 on the surface, q reduces to a given $F(x)$,which therefore satisfies the formula:

$$F(x)=\int_0^{\infty}\cos(mu)\,f(u)du$$

Cauchy later proved that:

$$F(x)=\frac{2}{\pi}\int_0^{\infty}\cos(mu)\,f(u)du$$

and, with this value of $f(m)$, $F(x)$ becomes:

$$F(x)=\int_0^{\infty}\int_0^{\infty}\cos(mx)\cos(mu)\,F(u)\;du\,dm$$

In this way Cauchy not only obtains the representation by means of the double Fourier integral of $F(x)$, but also the Fourier transform from $f(m)$ to $F(x)$ and the inverse transform

4. Il memoire sur les function

Among his many works having a purely algebraic character, the youthful Mèmoire sur les Function emerges. The importance of it depends above all on what the author here, continuing researches begun by Lagrange[14] and Ruffini[15], established some of the concepts that are at the basis of today's substitution theory and discovered some fundamental theorems. here are also numerous essential elements of the theory of determinants, which he arrived at starting from the product of mutual differences $A_{i-}A_k$ with $(i > k)$of n quantities. It should be noted that this paper, in the part dedicated to the determinants, presents impressive but casual points of contact with the Mèmoire sur un Système de Formules analytiques, published during the same year in the 16th century in the Journ. Ec. Pol, by another distinguished alumnus of the Polytechnic School, Binet[16]. He returned to the determined Cauchys after having found that the methods devised by Euler and Bèzout[17] by eliminating the unknown between two algebraic equations they lead to expressions of that form, so it was useful to improve the calculation methods. He then warns of its intervention in the integration of partial differential equations with constant coefficients. He finally got to grips with certain functionals when Jacobi[18] introduced them into

[14] Joseph-Louis Lagrange, (1736 - 1813), was an Italian mathematician and astronomer active, in his scientific maturity, for twenty-one years in Berlin and twenty-six in Paris. He is unanimously considered one of the greatest and most influential European mathematicians of the eighteenth century. His most important work is the Mécanique analytique, published in 1788, with which rational mechanics was conventionally born. In mathematics, he is remembered for his contributions to number theory, for being one of the founders of the calculus of variations, deducing it in the "Mecánique", for having outlined the foundations of rational mechanics, in the formulation known as Lagrangian mechanics, for the results in the field of differential equations and for being one of the pioneers of group theory. In the field of celestial mechanics he conducted research on the phenomenon of lunar libration and, later, on the movements of the satellites of the planet Jupiter; he investigated the problem of the three bodies and their dynamic equilibrium with the rigor of mathematical calculation. His pupils were Jean-Baptiste Joseph Fourier, Giovanni Plana and Siméon-Denis Poisson

[15] Paolo Ruffini (1765-1822) was a mathematician and physician Italian. He was a pupil of Luigi Fantini, a well-known expert in geometry , and of Paolo Cassiani, professor of analysis. ts name is linked to the Abel-Ruffini Theorem (probably conceived in 1803 or 1805), partial demonstration of the algebraic irresolubility of equations of degree higher than the fourth, through the theory of groups , and to the decomposition rule of polynomials . Furthermore, in 1809 , he published Ruffini's rule , an algorithm for dividing a polynomial into a variable by a binomial of first degree in the same variable. The algorithm allows to find both the quotient polynomial and the remainder polynomial . It is a simplified algorithm compared to the general one for the division of polynomials . In the philosophical field, he tried to demonstrate the immateriality of the soul.

[16]Jacques Philippe Marie Binet (1786 - 1856) was a mathematician and astronomer French . Binet entered the École polytechnique as a student in 1804 ; After graduating in 1806, he worked for the Ponts et Chaussées department, but the following year he returned to the École polytechnique as a repeater of descriptive geometry . Later he was professor of Mechanics, then inspector of studies. In 1823, he succeeded Jean-Baptiste Delambre in the chair of astronomy at the Collège de France . Like Cauchy , of whom he was a friend, Binet was a staunch Catholic and a supporter of the Bourbon family's claimant to the French throne . The government in July removed him from his duties at the École polytechnique, but he retained his posts at the Collège de France. His works on pure mathematics , mechanics and astronomy are published in the École polytechnique and in the Journal de Liouville.

[17] Étienne Bézout (1730 - 1783) was a mathematician French . Becoming a mathematician after reading Euler's works , Bézout taught in military schools, also becoming an examiner in competitions for admission to the Navy; he was assigned the task of writing a textbook for these courses, which, under the title *Cours de mathématiques à l'usage des Gardes du Pavillon et de la Marine* , was published in four volumes between 1764 and 1769, and later expanded, after becoming the successor of Charles Étienne Louis Camus as examiner of the Artillery Corps, as *Cours complet de mathématiques à l'usage de la marine et de l'artillerie*. This book was very popular, so much so that it was imitated and translated into several languages: for example, it was used at Harvard University.. In 1769 he became associated with the French Academy of Sciences . Bézout studied algebra , especially in the field of equations: in 1764 he published *Sur le degré des équations résultantes de l'évanouissement des inconnues* , a paper on the solution of systems of linear equations , defining by recurrence an amount equivalent to the determinant of the coefficients of the system. In his work *Théorie générale des équations algébraiques* , 1779, he proved Bézout's theorem , which states that two algebraic curves of degree (respectively) m and n intersect in general at $m \cdot n$ points. He also formulated theBézout identity for polynomials .

[18] Carl Gustav Jacob Jacobi (1804-1851) was a German mathematician and teacher. Jacobi in 1829 wrote his classic treatise on elliptic functions in which he faced the problem of "integrating the second order equations obtained from kinetic energy".
Jacobi was also the first mathematician to apply elliptic functions to number theory , in particular proving the theorem of the polygon number of Pierre de Fermat . The Jacobian theta function , applied in the studies of hypergeometric series , was named in his honor. His other important researches concerned differential equations, in particular the theory of the last multiplier , which is treated in his work Vorlesungen über Dynamik , edited by Alfred Clebsch (Berlin, 1866). He also wrote articles on the Abelian transcendents and his research in number theory , in which he was mainly concerned with completing the work of Gauss . He left behind a large number of manuscripts, some of which have been repeatedly published in the Journal de Crelle. His other works include Commentatio de transformatione integralis duplicis indefiniti in formam

the analysis. The contributions given by the supreme mathematician to algebra were many, unfortunately they cannot all be reviewed; but we cannot fail to focus on the very important proposition that teaches us to approximate the roots of an equation n complex coefficients $f(z) = o$; here is the statement:

$$f(x + iy) = P + iQ$$

This beautiful proposition was presented in 1831 at the Academy of Sciences of Turin, but the related memoir was never published in its entirety; the original demonstration given by Cauchy is an application of the calculation of residuals; he then replaced it with another elementary school. Serret[19], by inserting this theorem in his Cours d'Algèbre supèrieure with the proof given by C. Sturm and Liouville, disseminated its knowledge in school environments and then increased its value by showing how the fundamental theorem of algebra is a corollary.

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simpliciorem (1832), Canon arithmeticus (1839) and Opuscula mathematica (1846 - 1857). His Gesammelte Werke (1881 - 1891) was published by the Berlin Academy . Another important work is constituted by the Hamilton-Jacobi theory in the field of rational mechanics.

[19]Joseph-Alfred Serret (1829 - 1885) was a French mathematician and astronomer . After studying at the École polytechnique , in 1847 he obtained a doctorate in mathematics from the Faculty of Sciences of the University of Paris . The following year he was commissioned to teach higher algebra at the same university. Successor of Louis Poinsot at the French Academy of Sciences (1860) and professor of celestial mechanics at the Collège de France (1861), in 1873 he became a member of the Bureau des longitudes . He is best known for having developed, jointly with Jean Frédéric Frenet , the differential geometry formulas known as the Frenet-Serret formulas. He had the mathematical works of Gaspard Monge (1850) and Joseph Louis Lagrange (from 1867) published . The *rue Serret* , a small street of Paris in the fifteenth arrondissement , bears his name.

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