CAUCHY: THE THEORY OF GROUPS AND THE THEORY OF POLYHEDRA

Boniello Carmine
University of Salerno Fisciano (Italy)

Abstract
Cauchy was a mathematician who gave pure mathematics a great and important impetus; for fecundity and variety of production, he can be compared only to Euler: his writings, published during forty-seven years of continuous work, in separate volumes or in scientific collections, are about 789. To prevent this immense work from being lost, the Académie des Sciences in Paris began publishing the "Oeuvres complétes" as early as 1882, which is not yet finished. In this paper, we will examine two interesting innovations Cauchy made to mathematics of all time, the theory of groups, and the theory of polyhedra.

Keywords: Theory of groups, theory of polyhedra, mathematics, history

1. The theory of groups

The second contribution of fundamental importance due to Cauchy, in the opposite direction to the previous one, is that of combinatorial analysis. By penetrating to the core of Lagrange's method in the theory of equations, Cauchy was able to extract its essence, systematically elaborating the first foundations of group theory. Cauchy, who has always been a careful observer, saw the operations and their laws of combination under the symmetry of algebraic formulas, isolated them, and was led to the theory of groups. This theory was obtained by Cauchy starting from the theory of substitutions; in fact the French mathematician, elaborating the theory of substitutions in a long series of memoirs around 1840, developed it up to obtain the theory of finite groups. To have a precise degree of evolution and use of the theory of finite groups we must first of all indicate the main properties of a group of operations. We designate operations with capital letters $A, B, C, D, \ldots.$ so the subsequent execution order of two operations will be indicated by the correct position of the letters; so AB will mean that A was performed first, and B second; we see that AB and BA do not necessarily represent the same overall operation. For example, if A consists in adding 10 to a given number and B in dividing a given number by 10, AB applied to the number x will give

$$\frac{x+10}{10}$$

while BA will give

$$\frac{x+10}{10}$$

Vol. 6, No. 04; 2021

ISSN: 2456-3676

or

$$\frac{x+100}{10}$$

the two fractions are not the same; therefore AB and BA are two distinct operations. If the effects of two operations X and Y are the same, X and Y are said to be equal (or equivalent), and this equivalence is expressed by writing $X = Y$. Another fundamental notion is that of the possibility of association; if, for each group of three operations of a series U, V, W we have $(UV)W = U(VW)$, we say that the series is satisfied by the associative law; the symbol $(UV)W$ means that the UV operation is carried out first and that the operation W is carried out on the result obtained, while $U(VW)$ means that one begins by carrying out U and that on the result obtained the operation is carried out VW operation. Finally, the last fundamental notion is that of the identical operation, that is, operation I which leaves the matter on which it operates unchanged. Having acquired these notions, we can expose the simple postulate that defines what is called a group of operations.

It is said that a series of operations $I, A, B, C, \ldots X, Y, ..$ ... form a group if the first and fourth conditions we are about to expose are satisfied:

1. There is a combination rule applicable to any pair X, Y of operations (I) of the said series, such that the result XY of the combination $X, Y,$ carried out in this order, following the said combination rule, is an operation determined in a single way in the series.
2. For every triple operation X, Y, Z, in said series, the precedent pont is associative, that is $(XY)Z = X(YZ)$.
3. There exists a unique identity I in said series, such that for each operation X of it, $IX = XI = X$.
4. If X is any operation of said series, there is in this a unique operation, let's say X', such that $XX' = I$ (it can easily be proved that we also have $X's\,X = I$).

These postulates contain corollaries that can be deduced from other statements of the 1st and 4th, but in the above form they are grasped more easily. To give an example of the group, we can use the transpositions of letters; this may seem insignificant, but it is not, since these groups of transposition or substitution constitute the key, so long sought, of the possibility of algebraic resolution of the equations. There are exactly 6 orders of succession following which the three letters a, b, c can be arranged, that is: $abc, acb, bca, bac, cab, cba$. Let us take one of these orders, for example the first, abc, as the initial order: through which transpositions of letters do we have to go from this to the other five? To pass from abc to acb, just transpose or permute b and c; to represent this trade-in operation, we write (bc) that we read "b" in "c" and "c" in "b". To go from abc to bca, we change a to b, b to c, and c to a and write (abc). The order *abc* itself is taken from *abc* by null change, that is, a in a, b in b, c in c; is the substitution by identity, which we designate with *I*. Proceeding in the same way with the six orders:

Vol. 6, No. 04; 2021

ISSN: 2456-3676

$abc, acb, bca, bac, cab, cba,$

we get the corresponding substitutions

$I, (bc), (abc), (ab), (acb), (ac).$

The "combination rule" prescribed in the postulate above is the following: between these combinations, let us take any two, for example (bc) and (acb) and consider the effect of these two substitutions applied successively in the order stated, that is : firstly (bc) and secondly (acb); (bc) first of all changes b to c, then (acb) changes c to b; so then b is left as it was. Let us take the following letter, c, in (bc); by virtue of the operation (bc), changed to b which, by means of the operation (acb), it changed to a: thus, therefore, it changed to a. Continuing, let's see what it changed to: (bc) leaves it as it was, but (acb) changes it to c. Finally, the total effect of operation (bc) followed by operation (acb) is (ca), that we represent by writing:

$$(bc)\ (acb) = (ca) = (ac)$$

In the same way we can easily verify that:

$$(acb)\ (abc) = (abc)\ (acb) = I$$

$$(abc)\ (ac) = (ab)\ (bc)\ (ac) = (acb)$$

and so on for all possible couples. All this is summarized in the "multiplication table" of the group, which I write below, indicating, for space saving, the substitutions with the letters uppercase letters placed above each operation:

I	(bc)	(abc)	(ab)	(acb)	(axc)
I	A	B	C	D	E

To read this table we take a letter, for example C, in the column on the left and a letter for example D, in the top row; the letter found at the intersection of the row with the column:

	I	**A**	**B**	**C**	**D**	**E**
I	**I**	**A**	**B**	**C**	**D**	**E**
A	**A**	**I**	**C**	**B**	**E**	**D**
B	**B**	**E**	**D**	**A**	**I**	**C**
C	**C**	**D**	**E**	**I**	**A**	**B**
D	**D**	**C**	**I**	**E**	**B**	**A**
E	**E**	**B**	**A**	**D**	**C**	**I**

Corresponding is the result of the CD operation, that is $(ab)(acb)$. Thus $CD = A$, $DC = E$, $EA = B$ and so on. As an example, let's check the associative law for $(AB)\,C$ and $A\,(BC)$, which must be the same.

First of all:

$AB = C$

Therefore:

$(AB)\,C = CC = I$;

Secondly:

$BC = A$

Therefore:

$A = (BC) = AA = I.$

Likewise:

$A\,(DB) = AI = A;\ (AD)B = EB = A$

so therefore:

(AD) B = A (DB)

The total number of distinct operations in a group is called its order; here the group is of order 6. By examining the table, we derive several subgroups, for example:

	I
I	**I**

	I	**A**
I	**I**	**A**
A	**A**	**I**

	I	**B**	**D**
I	**I**	**B**	**D**
B	**B**	**D**	**I**
D	**D**	**I**	**B**

which are respectively of the order 1,2,3. In this way we arrive at one of the fundamental theorems proved by Cauchy: the order of a subgroup is a divider of the order of the group.

For any given order, the number of distinct groups, that is, having different multiplication tables, is finite, but it is not known (not even today) what this number is for any given order; we see therefore that from the beginning of a theory that at first glance seems as simple as a game of dominoes, we fall into unsolved problems. Having constructed the "multiplication table" of a group, let us no longer deal with the way in which it was deduced from the substitutions (if the substitutions served to create the table), and consider the table as defining an abstract group; to explain better, we do not give the symbols I, A, B, ... any other interpretation other than the one that implies the combination rule, such as $CD = A, DC = E$, etc. This abstract view is in current use today, but it was not in Cauchy's time; Cayley[1]introduced it in 1854, and until the first ten years of the twentieth century no completely satisfactory series of group postulates had been established. When we interpret the operations of a group as substitutions, or as rotations of a solid, or as any other branch of mathematics to which groups can apply, this interpretation is called the realization of the abstract group defined by the multiplication table. A given abstract group can have several realizations. For this reason, groups have a fundamental importance in modern mathematics: the abstract structure (summarized in the multiplication table) which serves as a basis for the same and same group is the essence of several theories that apparently have no relation to each other. , and thanks to an in-depth study of the properties of the abstract group, we get to know the theories in question and their reciprocal relationships by doing a single search instead of many. Limiting ourselves to just one example, let's say that the interplay of all the rotations that can be made to an icosahedron (regular solid of twenty faces) around its symmetrical axes, so that after each rotation the volume of the solid occupies the same space of first, it constitutes a group, and this group of rotations, when expressed in an abstract way, is the same group that occurs, following the exchange of the roots, when one wants to solve the general equation of the fifth degree. Furthermore, the same group does found in the theory of elliptic functions. This suggests to us that, although it is impossible to solve the fifth degree equation algebraically, the equation itself can be solved, and indeed it is, in terms of the functions we have spoken about. Finally, everything we have exposed can be geometrically represented by the rotations of the icosahedron in question. This beautiful unification is the work of Felix Klein[2] in his book on the icosahedron. Cauchy was one of the great promoters of the theory of substitution groups; after him, many works have been done on this subject, and the theory itself has largely developed thanks to the introduction of infinite groups (groups involving an infinity of operations which can be numbered, 1, 2, 3 ...); it has also been extended to groups of continuous movements. In the latter case, an operation of the group leads a body to assume another position by means of infinitesimal displacements (as small as we want), and not as we have seen for the icosahedral group in which the rotations moved the body of a finished quantity. This is but a category of infinite groups (here the terminology is not exact, but it is sufficient to bring out the important point, the distinction between discrete and continuous groups). Just as the theory of

[1] Arthur Cayley (1821 -1895) was an English mathematician who made a strong contribution to the growth of pure mathematics in the Anglo-Saxon world. Cayley was among the most prolific mathematicians of the nineteenth century. Even as a boy he enjoyed solving complex mathematical problems. He entered Trinity College in Cambridge at the age of 18 and excelled not only in mathematics but also in Greek, French, German and Italian. He later worked as a lawyer in London for 14 years , and produced around 250 mathematical research articles during this time. He then became a professor of pure mathematics at the University of Cambridge , and published about another 650. Cayley was responsible for introducing the product of matrices and the proof of the Hamilton-Cayley theorem, that is, of the fact that every square matrix is a root of its characteristic polynomial. He was also the first to express the general notion of a Group , as a set equipped with a binary operation that satisfies certain axioms (while previously group was synonymous with a group of permutations). He is also responsible for the theorem that bears his name , which states the isomorphism of each group with a group of permutations, and also the introduction of molecular graphs in chemistry .

[2] Felix Christian Klein (1849 - 1925) was a German mathematician . He is best known for his contributions to non-Euclidean geometry, to the links between geometry and group theory, and for some results on the theory of functions . He is also remembered for being the first descriptor of the geometric figure of hyperspace known as the Klein Bottle.

ISSN: 2456-3676

finite discrete groups is the basic structure for the theory of algebraic equations, so the theory of infinite continuous groups is of great use in the theory of differential equations, which are very important in mathematical physics.

2. The theory of polyhedra

Cauchy destined to rise to the highest reputation as an analyst began as an author under the guise of solver of geometric issues placed on the agenda at the time. The Academy had put a competition on the following problem: "Perfecting the theory of polyhedra in some essential point", and Lagrange had pointed out the subject to the young Cauchy as a fruitful field of research. In February 1811, Cauchy presented his first study on the theory of polyhedra, in which he answered in the negative to a question posed by Poinsot[3]: is it possible that there are regular polyhedra besides those of 4, 6, 8, 12 or 20 faces? In the second part of his memoir, Cauchy generalized Euler's formula, which the school textbooks of geometry in space give, concerning the number of edges A, arrows F and vertices S of a polyhedron:

$$A + 2 = F + S$$

This memory was printed; Legendre greatly appreciated her and encouraged Cauchy to continue; the advice was followed by the young man who published a second study in January 1812. The speakers were Legendre[4] and Malus[5]; the former expressed his enthusiasm and predicted great successes for the young author, the latter was much more reserved. Malus was not a professional mathematician, but an ancient official genius who had taken part in the Napoleonic campaigns in Germany and Egypt and who had made famous the accidental discovery of the polarization of light by reflection. It may be that his objections were considered by the young Cauchy in the category of those that can be expected from a stubborn physicist and therefore marked by a spirit of specious criticism. To demonstrate his most important problems, Cauchy had used the

[3] Poinsot Louis (1777 - 1859) French mathematician. A civil engineer by training, he later turned his interests and studies to abstract mathematics. Professor of analysis and mechanics at the École polytechnique (1809-16), on the death of J.-L. Lagrange he was called (1813) to succeed him in the Academy of Sciences. His scientific interests of the first period are reflected in the writings on the dynamics of solids (Théorie nouvelle de la rotation des corps, New theory of rotation of bodies, 1834) and above all in his fundamental contribution to that specific sector of mechanics which is called geometric mechanics : in 1803 the publication of the Élements de Statique (Elements of statics), founded on the theory of pairs of forces of which he can be considered the founder (→ Nagel, point of). Research in number theory and algebraic equations is also due to Poinsot.

[4] **Legendre** Adrien-Marie (1752 - 1833) French mathematician. He dealt with numerous fields of mathematics with significant and innovative results in the field of differential equations, function theory, number theory and applied mathematics. From 1775 he taught at the Military School of Paris and then at the École Normale. Member of the Académie des sciences of Paris since 1785, after the revolution he was part of the same academy, reconstituted with the name of Institute of Sciences and Arts. In 1790, on behalf of the constituent assembly, he was a member of the Committee of Weights and Measures, which defined the metric system. Legendre is responsible for notable results on elliptic integrals, the introduction of the least squares method and the conjecture on the distribution of prime numbers, according to which the number of prime numbers less than n approximates $n / \ln(n)$. His *Théorie des fonctions elliptiques et des intégrales eulériennes* crowned forty years of research on the traceability of the solution of numerous questions, both mathematical and physical, to the problem of integrating suitable functions that can be represented as arcs of curves, and offered a systematic study and classification of elliptic integrals marking the culmination of the first phase of development of the theory of integrals of algebraic functions, before the revolutionary innovations of NH Abel and KG Jacobi. In the *Théorie des nombres* the theory of congruences and the methods for identifying the integer roots of equations and systems of equations of first and second degree find expression; the priority in discovering such results was the subject of a controversy with Gauss. Legendre's work, partly overshadowed by that of great mathematicians of the time such as J.-L. Lagrange and A.-L. Cauchy, influenced mathematical training in Europe and the United States thanks in particular to the very successful geometry text (*Éléments de géométrie* , Elements of geometry, 1794), which had many editions and translations.

[5] Étienne-Louis Malus (1775 - 1812) was a physicist , engineer and mathematician French . He studied the phenomena of reflection, refraction and birefraction: in 1809 he discovered the phenomenon of the polarization of light by reflection.

ISSN: 2456-3676

"indirect method" that all beginners in geometry are familiar with; Malus took a stand against this method. The indirect method of proving a theorem consists in supposing the theorem false and in deducing an absurdity from this supposition, which, according to Aristotle's logic, leads us to affirm that the theorem is true. Cauchy was unable to refute the objection by providing direct evidence; Malus did not insist, but he was not convinced that Cauchy had proved anything. If in 1812 Malus failed to make Cauchy understand what the weakness was, he was avenged by Brouwer[6] in 1912 and even later, when he was able to prove to certain Cauchy's successors in mathematical analysis, that at least one point of this theory needed to be revised. . Aristotelian logic, as Malus tried to demonstrate to Cauchy, is not always a sure way of reasoning in mathematics. Encouraged by this success, Cauchy proposes to solve the other question of determining the conditions of equality or similarity of two polyhedra and then to see if a polyhedron is identified by its faces, and responds fully with another memory which, presented it even at the Institute of France, it was judged in the most authoritative way by a commission composed of Biot[7], Carnot[8] and Legendre. The conclusions to which Cauchy arrived in this work soon achieved great notoriety having been included by Legendre in the most recent editions of his gèomètrie. A meticulous examination of the works of Cauchy leads to reports many steps that teach new geometric and improvement propositions to some algebra applications to geometry; Champion of rigor He was reluctant to welcome the principle of continuity he taught how to avoid it in two "reports above Poncelet[9] Memories", which the Gergonne[10] hastened to publish in

[6] Luitzen Egbertus Jan Brouwer (Overschie , February 27, 1881 - Blaricum , December 2, 1966) was a Dutch mathematician , professor at the University of Amsterdam from 1912 to 1951 . He was the founder of the " school of intuition ", one of the main schools of the philosophy of mathematics of the nineteenth century . From an early age he showed an aptitude for mathematical research, and demonstrated a number of important theorems in topology . He studied in depth geometry , topology and the general theory of the relationships between logic and mathematics . In 1905 , at the age of 24, he published the philosophy text "Life, Art and Mysticism", which embryonicly marked the birth of his epistemological perspective of intuitionism in mathematics. In 1907 he obtained a Doctorate in Mathematics, and in 1912 he became a member of the Royal Academy of Arts and Sciences of the Netherlands . In those years he proved, among others, the fixed point theorem and the domain invariance theorem , fundamental in topology. In 1910 the great German mathematician David Hilbert , who highly esteemed him, began to help him obtain a permanent university chair, which he finally obtained in 1912 at the University of Amsterdam. In the following years, Brouwer developed his intuitionist theses, often in direct polemic against Hilbert himself, who was instead the main representative of the formalist approach to the foundations of mathematics.

[7] Jean-Baptiste Biot (1774 - 1862) was a physicist and mathematician French . He is best known for studying the relationship between electric current and magnetism (he and Félix Savart are entitled to Biot-Savart's Law , which provides the value of the magnetic field generated at a point in space by an electric current as a function of distance from the conductor) and the phenomenon of the rotation of the plane of polarization of light in the passage through solutions of chemical compounds. Based on these results, Biot used the saccharimeter to determine the nature and amount of sugars in a solution. He was the first to discover the characteristic optical properties of mica ; due to this, biotite , a mica mineral, also bears his name. In 1800 he was appointed professor of physics at the *Collège de France* and was elected a member of the French Academy of Sciences at the age of 29. In 1804 he made with Joseph Louis Gay-Lussac the first scientific ascent in a hot air balloon to an altitude of 5,000 meters, conducting one of the first surveys on the composition of the Earth's atmosphere . He held the chair of astronomy at the Faculty of Science from 1809 to 1816 and from 1826 to 1848 . It is also called a small lunar crater . He received the *Rumford* Medal in 1840 .

[8] Nicolas Léonard Sadi Carnot (1796 - 1832) was a physicist, engineer and French mathematician. To him must be very important contributions to theoretical thermodynamics, among these, the theorisation of what will be called the carnot machine, the carnot cycle and the carnot theorem, whose statement states that any thermodynamic machine that works between two heat sources A different temperature, it must necessarily have a performance that cannot exceed that of the carnot machine.

[9] Jean-Victor Poncelet (1788 - 1867) was a French mathematician and engineer who made many contributions to the development of projective geometry . Born into a poor family in Metz , Poncelet wins a scholarship to high school and then to the *École Polytechnique* where he studies

the Annales of him. No lesser value have the notes he attached to a "report over a memory of Amiot[11]" presented to the Institute of France concerning the focal theory of the quadrics

3. Conclusions

In light of the above, we can certainly affirm that only with Cauchy the substitution groups have penetrated today into the modern theory of the atomic structure so we can say that the theory of groups finds a wide diffusion both among elementary operations and among applications in higher mathematics. Cauchy had not foreseen these applications of a theory that he had established solely for the attraction it exerted on him, just as he had not foreseen its application to the solution of algebraic equations. Today, this elementary theory, although complicated, is of capital importance in many fields of pure and applied mathematics, from the theory of algebraic equations to geometry and the theory of atomic structure; to mention only one of its applications, we find it at the basis of crystal geometry, and its further developments, from the analytical side, extend far beyond in higher mechanics and in the modern theory of differential equations. No less interesting was his dissertation on the theory of polyhedra. In light of the foregoing, we can highlight that one of the often overlooked elements is the fact that, as in many other sciences, also in mathematics, new theories and new points of view have historically established themselves through discussions, polemics and sometimes decided contrasts concepts and methods

Bibliography

1. Belhoste B.: Augustin-Louis Cauchy. A Biography New York, 1991.
2. Belhoste B.: Cauchy. Un mathématicien légitimiste au XIXe siècle Paris, 1985.
3. Bliss Gilbert A.: *The reduction of singularities of plane curves by birational transformation*, "Bulletin of the American mathematical society", 29, 1923, pp. 161-168.
4. Bogolyubov A. N.: Augustin Cauchy and his contribution to mechanics and physics (Russian), Studies in the history of physics and mechanics, 'Nauka' Moscow, 1988, 179-201.

under the supervision of Gaspard Monge . In 1810 he entered the military engineering college. Participate as a lieutenant in the Russian campaign with Napoleon . He is believed dead and left at the Battle of Krasnoi , then imprisoned by the Russians in Saratov and repatriated to France in 1814 . During his imprisonment, he studied projective geometry and wrote some drafts of the book " *Applications d'analyse et de géométrie* " which would be published in two volumes in the years 1862 - 1864 . Study the conic sections and develop the principle of duality . Upon his repatriation, he became professor of mechanics at the *école d'application* in 1825 . In his lectures he coins the term *fatigue* to describe the state of materials subjected to stress . He is particularly interested in the design of turbines and water wheels , and designs a Francis turbine in 1826 (the first model, however, was built in 1838). In his book " *Industrial Mechanics* " (1829) he studies work and kinetic energy. Some attribute it to the definition of work as a product of force and translation . Poncelet left Metz in 1835 and became a professor of mechanics at the Sorbonne in 1838 . Since 1848 , he has commanded the *École Polytechnique* with the rank of general. He then retired in 1850 to devote himself to mathematical research. An asteroid was dedicated to him , 29647 Poncelet . In addition, a prize for physics was established in his name, which among others, was awarded to Rudolf Emanuel Clausius.

[10] Joseph Diaz Gergonne (1771 - 1859) was a French mathematician , known for his contributions to projective geometry and for being the founder of the periodical *Annales de mathématiques pures et appliquées* , the first magazine dedicated to maths.

[11] Alphonse Amiot (1812-1865), noto anche come Antoine Désiré Alphonse Amiot, matematico francese.

5. Bottazzini U.: *Il calcolo sublime. Storia dell'analisi matematica da Euler a Weierstrass*, Torino, Gray Jeremy J.: *Algebraic geometry between Noether and Noether. A forgotten chapter in the history of algebraic geometry*, "Revue d'histoire des mathématiques", 3, 1997, pp. 1-48.
6. Cutland N. - Kessler C.: , E Kopp and D Ross, On Cauchy's notion of infinitesimal, British J. Philos. Sci. 39 (3) 1988, 375-378.
7. Dahan-Dalmedico A.: L'intégration des équations aux dérivées partielles linéaires à coefficients constants dans les travaux de Cauchy 1821-1830, Études sur Cauchy (1789-1857), Rev. Histoire Sci. 45 (1), 1992, 83-114.
8. Dahan-Dalmedico A.: La mathématisation de la théorie de l'élasticité par A L Cauchy et les débats dans la physique mathématique française (1800-1840), Sciences et Techniques en Perspective 9, 1984-85, 1-100.
9. Dahan-Dalmedico A.: Les travaux de Cauchy sur les substitutions: étude de son approche du concept de groupe, Arch. Hist. Exact Sci. 23 (4), 1980/81, 279-319.
10. Dubbey J. M.: Cauchy's contribution to the establishment of the calculus, Ann. of Sci. 22 1966, 61-67.
11. Fisher G.: Cauchy's variables and orders of the infinitely small, British J. Philos. Sci. 30 (3), 1979, 261-265.
12. Fisher G.: Cauchy and the infinitely small, Historia Mathematica 5, 1978, 313-331.
13. Freudenthal, H. Did Cauchy plagiarise Bolzano, Archive for History of Exact Sciences 7, 1971, 375-392.
14. Gario P.: Cauchy's theorem on the rigidity of convex polyhedra (Italian), Archimede 33 (1-2), 1981, 53-69.
15. Gilain C.: Cauchy et le cours d'analyse de l'École Polytechnique, Bull. Soc. Amis Bibl. École Polytech. (5), 1989, 1-145.
16. Grabiner J. V.: The origins of Cauchy's theory of the derivative, Historia Math. 5 (4), 1978, 379-409.
17. Grabiner J. V.: Who gave you the epsilon? Cauchy and the origins of rigorous calculus, Amer. Math. Monthly 90 (3), 1983, 185-194.
18. Grattan-Guinnes I.: Bolzano, Cauchy and the 'new analysis' of the early nineteenth century, Archive for History of Exact Sciences 6, 1970, 372-400.
19. Grattan-Guinness I.: On the publication of the last volume of the works of Augustin Cauchy, Janus 62 (1-3), 1975, 179-191.
20. Grattan-Guinness I.: The Cauchy - Stokes - Seidel story on uniform convergence again: was there a fourth man?, Bull. Soc. Math. Belg. Sér. A 38, 1986, 225-235.

ISSN: 2456-3676

21. Gray J.: , Cauchy - elliptic and abelian integrals, Études sur Cauchy (1789-1857), Rev. Histoire Sci. 45 (1) ,1992, 69-81.
22. Gui Z.L.- Zhao D. F., Cauchy : an outstanding mathematician (Chinese), J. Central China Normal Univ. Natur. Sci. 23 (4), 1989, 597-60
23. Hawkins T.: Cauchy and the spectral theory of matrices, Historia Mathematica 2 (1975), 1-29.
24. Hawkins Thomas W.: *New light on Frobenius' creation of the theory of group characters*, "Archive for history of exact sciences", 12, 1974, pp. 217-243.
25. Koetsier T.: Cauchy's rigorous calculus : a revolution in Kuhn's sense?, Nieuw Arch. Wisk, 1987, 335-354
26. Montesinos J. M.: *Classical tessellations and three-manifolds*, Berlin-London, Springer, 1987.
27. Ostrowski A. M.: *Über den ersten und vierten Gaussschen Beweis des Fundamentalsatzes der Algebra*, Berlin, Springer, 1933.
28. Purkert W.: *Ein Manuskript Dedekinds über Galois-theorie*, "NTM. Schriftenreihe für Geschichte der Naturwissenschaften, Technik und Medizin", 13, 1976, pp. 1-16. Rider R. E.: *A bibliography of early modern algebra, 1500-1800*, Berkeley, Office for History of Science and Technology, University of California, 1982.
29. Rider R. E., *A bibliography of early modern algebra, 1500-1800*, Berkeley, Office for History of Science and Technology, University of California, 1982.
30. Scholz E.: Symmetrie, Gruppe, Dualität, zur Beziehung zwischen theoretischer Mathematik und Anwendungen in Kristallographie und Baustatik des 19. Jahrhunderts, Basel, Birkhäuser, 1989.
31. Smithies F.: The background to Cauchy's definition of the integral, Math. Centre Tracts 149, Amsterdam, 1982, 93-100.
32. Struik D. - Struik R.: Cauchy and Bolzano in Prague, Isis 11, 1928, 364-366.
33. Taton R.: Sur les relations scientifiques d'Augustin Cauchy et d'Évariste Galois, Rev. Histoire Sci. Appl. 24 (2), 1971, 123-148.
34. Valson C A: La vie et les travaux du baron Cauchy Paris, 1868.