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Cauchy an Innovative Mathematician: the Fundamentals of Infinitesimal Calculus, the Theorem of Finite Increments and the Functions of an "Imaginary" Variable

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Abstract
Calculus is the founding branch of mathematical analysis that studies the "local behavior" of a function through the notions of continuity and limit, used in almost all fields of mathematics and physics and science in general. In the article we wanted to highlight through Cauchy the objectives of infinitesimal analysis expand and include complex analysis. In the second part of the work we concentrated on the finite increment theorem. It is one of the classical theorems of Mathematical Analysis, whose importance is justified by the fact that Lagrange's theorem turns out to be a trivial consequence. Finally, we will highlight its contribution to the functions of an imaginary variable: it is a contribution of fundamental importance for mathematics scholars of all times.

Keywords: Mathematics, infinitesimal calculus, history, theorem

1. The fundamentals of infinitesimal calculus

Certainly one of Cauchy's greatest merits is that he presented the principles of infinitesimal analysis in a fully rigorous form, both as regards real variables and as regards complex variables. The first teachers of the Ecole Polytecnique had set a precedent according to which not even the greatest mathematicians disdained writing manuals of all levels, and Cauchy[1] also faithfully followed this tradition.

In three books he gave the elementary exposition of the infinitesimal calculus the appearance and character that it still has today, they were *il Cours d'analyse dell'Ecole Polytecnique* (1821), the *Résumé des lecons sur le calcul infinitésimal* (1823), and the *leçons sur le calcul différentiel* (1829). Calculus was the subject of the second part of Cauchy's course, which he published in 1823 Résumé des lecons sur le calcul infinitésimal. This volume contains the definitions of derivative, differential and integral in terms of limits, which Cauchy had been presenting for some years to the students of the École Polytechnique and which have since become classical. Cauchy was well aware of the theoretical novelty of his ideas on the foundations of calculus, radically opposed to the Lagrangian ones to which his colleague in the teaching of analysis at the École, Ampère, adhered. In the warning with which the Résumé des lecons sur le calcul infinitésimal opens, he explicitly wrote: "The methods I have followed differ in several respects

[1] Cauchy Augustin Louis (1789 - 1857) was a French mathematician and engineer. He initiated the project of the formulation and rigorous demonstration of the theorems of infinitesimal analysis based on the use of the notions of limit and continuity. He also made important contributions to the theory of complex variable functions and to the theory of differential equations. The systematic nature and level of these works place him among the fathers of mathematical analysis.

from those which are exhibited in works of the same genre. My main purpose was to reconcile the rigor, which I had set myself as an indispensable norm in my Cours d'analyse, with the simplicity that comes from the direct consideration of infinitesimals". In fact, as in the Cours, also in the Résumé, once the concept of infinitesimal was defined by means of that of limit, he then made extensive use of infinitesimals in proofs. In the introductory pages, Cauchy also denounced the open insufficiency of the "Method of series" and Taylor's series to found the calculus, contrary to what Lagrange[2] had stated in Theorie; and in this regard he wrote: "Despite all the respect that one owes to a great authority - observed Cauchy in this regard - most of the surveyors today agree in recognizing the uncertainty of the results to which one can be brought using the divergent series". In particular, in several cases the Taylor series said Cauchy: "it seemed to provide the development of a convergent series function, although the sum of the series differs in an essential way from the proposed function". This was the astonishing result he had achieved by considering the example of the function:

$$f(x) = e^{\frac{-1}{x^2}}$$

whose directives are all null for $x = 0$. Cauchy had presented his counterexample of "Taylor's theorem" to the Academie in 1822 and then in a note that appeared the same year in «Bulletin de la Société Philomatique», which provoked an immediate response in the same newspaper written with every evidence by the most convinced follower of Lagrange, Pòisson[3]. For his part, Cauchy did not reply directly to Poisson's arguments, limiting himself to reiterating his own convictions in the pages of the Résumé. The same theoretical structure of the volume reflected a conception of calculus opposite to that of Lagrange and Poisson. While in the Théorie, in fact, Taylor's series appeared in the opening pages as the basis of the whole construction of the derivative function theorem, it was treated by Cauchy in the last lessons of the Résumé, after the introduction of the concepts of derivative and definite integral. As he explained in the warning, in fact, he was "*forced to postpone Taylor's formula to the integral calculus, this formula being allowed only as long as the series contained therein is limited to a finite number of terms and completed with an integral defined ".* After presenting the concepts of limit, infinitesimal and continuity of a function in the same terms used in the *Cours d'analyse*, Cauchy in the third lesson of the Résumé introduced the notion of derivative of a function $f(x)$. He considered the ratio incremental:

[2] Lagrange Joseph-Louis, (1736 –1813), was an Italian mathematician and astronomer active, in his scientific maturity, for twenty-one years in Berlin and twenty-six in Paris. He is unanimously considered among the greatest and most influential European mathematicians of the eighteenth century; his innovative contributions to mathematical physics are also noteworthy. His most important work is the Mécanique analytique, published in 1788, with which rational and analytical mechanics are conventionally born. In mathematics, he is remembered for his contributions to number theory, for being one of the founders of the calculus of variations (deducing it, in his "Mécanique analytique", through that theoretical formulation of rational mechanics known as Lagrangian mechanics), for the results in the field of differential equations and infinitesimal analysis, as well as for having been one of the pioneers of group theory and classical field theory. In the field of celestial mechanics, he conducted research on the phenomenon of lunar libration and, later, on the movements of the satellites of the planet Jupiter; he investigated, with the rigor of mathematical calculation, the problem of the three bodies and their dynamic equilibrium; he also devoted himself to studies of natural sciences. His pupils were Jean-Baptiste Joseph Fourier, Giovanni Plana and Siméon-Denis Poisson.

[3] Poisson Siméon-Denis (1781 - 1840) was a French mathematician, physicist, astronomer and statistician. Of modest origins, he was encouraged to study and entered the École Polytechnique in Paris in 1798. He became a teacher of this school also thanks to the support of Laplace and in 1806 he succeeds Fourier. In 1816 he obtained a chair of mechanics at the Sorbonne and was elected to the Academy of Sciences in Paris.

$$\frac{\Delta y}{\Delta x} = \frac{f(x+a) - f(x)}{a}$$

where$a$ was an infinitesimal and studied its limit as a tends towards zero. In this regard, Cauchy reiterated: *"This limit, if it exists, has a certain value for each particular value of x, but varies with x"*. Cauchy gave it the name of derived function$f'(x)$, preserving the terminology and the Lagrangian notation. In a similar way, Cauchy defined the concept of differential by means of an appropriate limit $dy$of a function $= f(x)$. Then showing that equality $dy = f'(x)dx$ allowed to write the derivative $f'(x)$ as the quotient of the differentials$\frac{dy}{dx}$, he made rigorous a consolidated practice in mathematics since the time of Leibniz[4]. Alongside that of derivative, the other fundamental notion of calculus was that of integral. Traditionally integration had been considered by mathematicians to be the inverse operation of derivation; in Lagrange's words, the search for a "primitive" function $F(x)$ such that its "derivative" $F'(x)$ was equal to the starting function $f(x)$. The fundamental theorem of calculus then allowed us to introduce the definite integral:

$$\int_b^a f(x)\,dx$$

using the formula:

$$\int_b^a f(x)\,dx = F(a) - F(b) \qquad \text{With} \qquad F'(x) = F(x)$$

Cauchy in his lectures at the École Polytechnique consciously departed from a well-established practice: *"It seemed necessary, he wrote in the warning to the Résumé, to generally demonstrate the existence of primitive integrals or functions, before making their different properties known. For this purpose, it was essential first of all to establish the notion of integral taken within given limits or defined integral "*. He therefore considered a continuous function $f(x)$in an interval$[xo, X]$ e an arbitrary subdivision of this interval into$n$ parts. The sum S of the products:

$$S = (X_1 - X_2)\,f(X_0) + (X_2 - X_1)\,f(X_1) + \ \ldots\ldots + (X - X_{n-1})\,f(X_{n-1})$$

it depended on the number of parts into which the interval had been divided and on the way of subdivision. However, it was not difficult to show that if the number of subdivision intervals became "very large", and therefore the amplitude of each interval very small, the subdivision mode became irrelevant on the value of S which, as n increases indefinitely; and about this Cauchy wrote: "*It will end up being sensibly constant or, in other words, it will end up reaching a certain limit, which will depend solely on the form of the function $f(x)$and on the values of the extremesx_0, X attributed to the variable*". This limit is what is called a definite integral, Cauchy concluded. Considering then variable$x$, also the sum $S(x)$was a continuous function of$x$, which had as derivative$f(x$; in other words it was, up to a constant, the indefinite integral of $f(x)$. Of

[4] Leibniz Gottfried Wilhelm von (1646-1716) was a philosopher, mathematician, scientist , logician, theologian, linguist, linguist, diplomat, jurist, historian, German magistrate. Considered the forerunner of computer science, neuroinformatics and automatic computing, he was the inventor of a mechanical calculator called the Leibniz machine; moreover, some areas of his philosophy opened numerous glimpses on the dimension of the unconscious that only in the twentieth century, with Sigmund Freud, will we try to explore.

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course, things got complicated when $f(x)$ was discontinuous in a finite number or in an infinity of points, a problem that will soon prove to be of the utmost importance in research on trigonometric series. Cauchy's insistence on the need to *"generally prove the existence"* of indefinite integrals revealed, together with a great theoretical novelty, the central role that questions of existence were assuming in mathematics; among these was the problem of the existence of solutions for the differential equations, of which:

$$dy = f(x)\,dx$$

which led to the concept of an indefinite integral:

$$\int f(x)\;dx$$

which represented a first particular case. The theory of differential equations was the subject of second-year analytical courses for students of the École Polytechnique. Cauchy first dealt with the classical cases of the "exact" integration of differential equations in two variables of the form:

$$Pdx + Qdy = 0$$

and then went on to discuss the first cases of the so-called *"Cauchy problem"*,
considering the integration of a differential equation of the form:

$$dy = f(x,y)dx$$

whereby the solution $y(x)$would satisfy a given initial values$x_0 y_0$, (around which$f(x,y)$ and the respective derivatives were supposed to be continuous). Revealing a great theoretical unity between the first and second year courses, the demonstration of the existence and uniqueness of the solution in a neighbourhood of the point x_0 , y_0 was conducted by Cauchy with methods quite similar to those introduced in the definition of the concept of integral, where once again the notion of limit proved to be technically essential. As requested by the Conseil de Perfectionnement, Cauchy was preparing to publish his second year lectures as well, and the first printed sheets had to circulate among colleagues and students in 1824, before an abrupt and definitive interruption of publication. The reading of Cauchy's lectures did not in fact impress the members of the Conseil: the topics were treated in a too abstract and difficult way, *"they could be suitable for the Faculty of Sciences but not for the École Polytechnique"* reads the minutes of a meeting. Then repeatedly the students complained to the management of the École for the great difficulty in following Cauchy's lessons. Faced with the renewed requests of the Conseil to modify the contents and teaching methods, after having tried to defend his autonomy as a teacher in the choice of methods, Cauchy was forced to announce in November 1825 that *"in order to comply with the wishes of the Conseil he will propose himself more to give, as he has done so far, perfectly rigorous demonstrations "* in his lectures. Hence also Cauchy's loss of interest in continuing the publication of a text that could no longer reflect his deepest convictions: what is now considered a decisive first step towards modern rigor in mathematics was reproached to Cauchy by his contemporaries. as a *"luxury of analysis"* or even a *"lack of clarity"* inadvisable, if not counterproductive, for the students of the École Polytechnique

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2. ***The theorem of finite increments***

Another Cauchy's contribution to mathematics was the finite increment theorem.

This theorem is of fundamental importance for the "differential calculus", it states that: *"If $f(x)$ and $g(x)$ are two continuous functions in the closed interval $[a, b]$ and differentiable internally to it, and if the derivative $g'(x)$ never vanishes, there is at least one point c, inside to the closed and limited interval $[a, b]$, such that it is"*:

$$\frac{f(b) - f(a)}{\varphi(b) - \varphi(a)} = \frac{f'(c)}{\varphi'(c)}$$

first of all it should be noted that it certainly result $\varphi(b) - \varphi(a)$different from 0 why it turned out $\varphi(b) - \varphi(a)$, by Rolle[5]'s theorem, the derivative $\varphi'(x)$ it should cancel itself out at least in one point, against the hypothesis made. Given this, we form the function:

$$g(x) = f(x) + k\varphi(x)$$

where k is a constant that we will determine so that the function $g\,(x)$ verify the condition $g(a) = g(b)$, that is:

$$f(a) + k\varphi(a) = f(b) + k\varphi(b)$$

from which:

$$k = -\frac{f(b) - f(a)}{\varphi(b) - \varphi(a)}$$

for such a choice of k one can apply Rolle's theorem to $g\,(x)$ and hence there exists a point "$c$inside $[a, b]$ for which it is:

$$g'(c) = f'(c) + k\varphi(x) = 0$$

that is:

$$\frac{f'(c)}{\varphi'(c)} = -k$$

where

$$\varphi'(c) \neq 0$$

from which, bearing in mind the value of$k$, found by means of the formula previously obtained, we obtain:

$$\frac{f(b) - f(a)}{\varphi(b) - \varphi(a)} = \frac{f'(c)}{\varphi'(c)} \qquad (a < b < c >)$$

[5] Michel Rolle (1652 - 1719) was a French mathematician. He is mainly known for having formulated, in 1691, in the particular case of real one-variable polynomials, a first version of the theorem that bears his name.

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3. *The functions of an "imaginary" variable*

A good half of the Cours d'analyse was devoted to "imaginary" quantities, which Cauchy[6] introduced in a completely formal way as particular "symbolic expressions", that is, "combinations of algebraic signs which mean nothing in themselves" and, taken literally, "Are inaccurate or have no meaning, but from which exact results can be deduced". This attitude, both formal and pragmatic, had been adopted by Cauchy since his Mémoire sur les intégrales définies of 1814, where the "passage from the real to the imaginary" allowed him to calculate a large number of integrals and highlight the limitations, which Euler[7] and Lagrange[8] escaped, which in general prevented the reversal of the order of integration in a double integral. In Mémoire itself, many of the fundamental concepts of the theory of the functions of a complex variable were expressed in embryonic form, which Cauchy would then develop from the 1920s for the rest of his life in a disordered and torrential production of notes, memories, pamphlets. and communications presented in the weekly sessions of the Accadémie. In the Cours d'analyse he limited himself instead to familiarizing the reader with the basic notions: algebraic operations with "imaginary" quantities (or complex numbers, as we say today), limits and infinite series, the notions of complex variable and of function of such a variable, induced as simple generalizations of the analogous concepts already defined in the real case. Cauchy's pragmatic attitude towards complex numbers and variables was then very common among French mathematicians; although of an uncertain theoretical nature, "imaginary" quantities were increasingly used in research, both in the evaluation of definite integrals and in the integration of differential equations, and Poisson[9] had shown in several articles that appeared in the "*Journal de l'École Polytechnique*"

[6] Cauchy Augustin Louis (1789 - 1857) was a French mathematician and engineer. He initiated the project of the formulation and rigorous demonstration of the theorems of infinitesimal analysis based on the use of the notions of limit and continuity. He also made important contributions to the theory of complex variable functions and to the theory of differential equations. The systematic nature and level of these works place him among the fathers of mathematical analysis.

[7] Euler Leonhard, known in Italy as Euler (1707 - 1783) was a Swiss mathematician and physicist. He is considered the most important mathematician of the eighteenth century, and one of the greatest in history. It is known to be among the most prolific of all time and has made historically crucial contributions in various areas: infinitesimal analysis, special functions, rational mechanics, celestial mechanics, number theory, graph theory. Pierre Simon Laplace seems to have said "Read Euler; he is the teacher of us all". Euler was undoubtedly the greatest supplier of "mathematical denominations", offering his name to an impressive amount of formulas, theorems, methods, criteria, relations, equations. In geometry: the circle, the line and Euler's points relative to the triangles, plus the Euler-Slim relation, which concerned the circle circumscribed to a triangle; in number theory: Euler's criterion and Fermat-Euler's theorem, Euler's indicator, Euler's identity, Euler's conjecture; in mechanics: Euler angles, Euler critical load (due to instability); in the analysis: the Euler-Mascheroni constant, the Euler gamma function; in logic: the Euler-Venn diagram; in graph theory: (again) the Euler relation; in algebra: Euler's method (relative to the solution of the fourth degree equations), Euler's theorem; in differential calculus: Euler's method (concerning differential equations).

[8] Lagrange Joseph-Louis, (1736 –1813), was an Italian mathematician and astronomer active, in his scientific maturity, for twenty-one years in Berlin and twenty-six in Paris. He is unanimously considered among the greatest and most influential European mathematicians of the eighteenth century; his innovative contributions to mathematical physics are also noteworthy. His most important work is the Mécanique analytique, published in 1788, with which rational and analytical mechanics are conventionally born. In mathematics, he is remembered for his contributions to number theory, for being one of the founders of the calculus of variations (deducing it, in his "Mécanique analytique", through that theoretical formulation of rational mechanics known as Lagrangian mechanics), for the results in the field of differential equations and infinitesimal analysis, as well as for having been one of the pioneers of group theory and classical field theory. In the field of celestial mechanics, he conducted research on the phenomenon of lunar libration and, later, on the movements of the satellites of the planet Jupiter; he investigated, with the rigor of mathematical calculation, the problem of the three bodies and their dynamic equilibrium; he also devoted himself to studies of natural sciences. His pupils were Jean-Baptiste Joseph Fourier, Giovanni Plana and Siméon-Denis Poisson.

[9] Poisson Siméon-Denis (1781 - 1840) was a French mathematician, physicist, astronomer and statistician. Of modest origins, he was encouraged to study and entered the École Polytechnique in Paris in 1798. He became a teacher of this school also thanks to the support of Laplace and in 1806 he succeeds Fourier. In 1816 he obtained a chair of mechanics at the Sorbonne and was elected to the Academy of Sciences in Paris.

between 1820 and 1823 the usefulness and modalities of the use of "imaginaries" in integration, in particular in the computation of definite integrals of "functions that pass through infinity". The problem of the integration "between imaginary limits" of a function of a complex variable was addressed in all its generality by Cauchy in a pamphlet published in 1825, which in the opinion of many constitutes one of the most important articles in the history of mathematics. To understand in the same definition the integrals taken between real limits and those taken between imaginary limits »Cauchy proposed to consider the latter integrals as limits, in a similar way to how he had defined the former in the Résumé. If, as was then implied in Cauchy's words but will soon become usual, the "imaginary" extremes of integration are interpreted as points of the complex plane, one immediately realizes that there are infinite continuous "paths" in the plane that join the two points:

$$a+\sqrt{-1}b \qquad\qquad c+\sqrt{-1}d$$

Cauchy then proved that the value of the integral derived from the two points
it was the same for any two paths, under suitable regularity hypotheses for the integrating function in the part of the plane between them. This fundamental *"integral theorem"* which today bears his name is one of the most profound propositions of the whole theory of functions of a complex variable. The next step in his memory of 1825 was for Cauchy to consider functions that became infinite within the domain of integration, which presented a "pole" as it is called today.

For such cases Cauchy introduced the concept of "residual of the function" in the *"singular"* point, defined as the integral of the function taken along an infinitesimal closed path including the infinitesimal point within it and then developed the "computation of residuals" as a sort of calculation in his opinion *"analogous to the infinitesimal calculus"*. The residue theory appeared to Cauchy far more important than his own *"Integral theorem"*. Numerous of the articles that filled the *Exercires des mathématiques*, a sort of real personal magazine that Cauchy then began to publish in periodic issues. Cauchy's copious and disordered scientific production in those years showed that he had gone far beyond his contemporaries in the study of complex analysis, even if one could speak more of single results, some of them very profound, than of a real organic theory in its various parts. It certainly did not contribute to increasing the clarity, the variety of names and special symbols that Cauchy liked to introduce into his works or the diversity of intentions that the French mathematician had in view from time to time. The fundamental ideas on integration in the complex field, which Cauchy had published in 1825, had however been known for some time also to Gauss[10], who in 1811 had written about them to his friend Bessel[11].

[10] Gauss Johann Friedrich Carl (1777-1855) was a German mathematician, astronomer and physicist, who made decisive contributions in mathematical analysis, number theory, statistics, numerical calculus, differential geometry, geodesy, geophysics, magnetism, electrostatics, astronomy and optics. Sometimes referred to as "the Prince of mathematicians" (Princeps mathematicorum) as Euler or "the greatest mathematician of modernity" (as opposed to Archimedes, considered by Gauss himself as the greatest mathematician of antiquity), he is counted among the most important mathematicians in history having contributed decisively to the evolution of the mathematical, physical and natural sciences. He defined mathematics as "the queen of the sciences".

[11] Bessel Friedrich Wilhelm (1784–1846) was a German mathematician, astronomer and geodesist, systematizer of Bessel's functions (which, despite the name, were discovered by Daniel Bernoulli).

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