

**Determining and Correctly Understanding the Damping Coefficient in Harmonic Oscillation Reduces the Calculation Volume and Modifies Knowledge for the Designer**

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**Abstract**

The damping coefficient in engineering is an important issue. Currently, science uses computers to determine this coefficient. With a considerable amount of input data, solving is complicated. Therefore, manual calculation is an insurmountable challenge. This paper presents a valuable theory to determine the damping coefficient, simplifying the data entry process for processing through the computer. Thereby, the method saved effort and time.

Determining the damping coefficient in theoretical mechanics is very difficult. So far, it has often been determined experimentally. Because all measurement methods have errors, the determination is even more difficult. When the number of objects that need to measure damping parameters in practice is large, the calculation and processing process takes more effort.

The mathematical and physical theory used in the article has been around for a long time. However, these theories were applied to the problem in a completely new way and solved the problem fundamentally.

The larger the calculation volume, the more damping included factors and the more meaningful this theory is. The approach also clarifies that the damping coefficient is a parameter that depends on stiffness and mass rather than an independent parameter in harmonic oscillation. This additional knowledge also contributes to understanding and applying this parameter's mathematical and physical nature in science and practice. At the same time, it also opens up completely new research directions.

**Keywords:** Physics, math, damping coefficient, oscillate, breakthrough.

**Introductions**

Mechanical oscillations, including harmonic oscillations, are interesting, researched, and solved by many authors. Harmonic oscillations are defined as parameters that are bound to each other according to a general equation.

$$m\ddot{X} + c\dot{X} + kX = F \quad ( )$$

In which:  $m$  – mass,  $c$  – damping coefficient,  $k$  – stiffness,  $\ddot{X}$  – acceleration,  $\dot{X}$  - velocity,  $X$  - distance.

The salient feature of this oscillation is its continuous, harmonic nature. Parameters  $c$ ,  $m$ , and  $k$  are measured and put into the calculation. Measuring and processing take much effort when the number of objects surveyed increases. To make matters more clear, consider the following example:

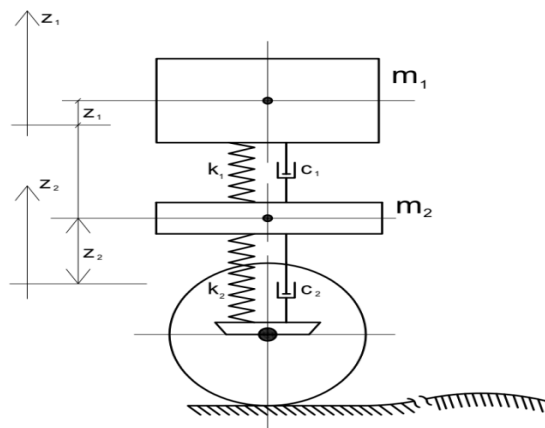


Figure1. Model of passenger wagon.

$$M\ddot{Z} + C\dot{Z} + KZ = F \quad (1)$$

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \text{ (a). Mass Matrix.}$$

$$K = \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 + K_2 \end{bmatrix} \text{ (b). Stiffness matrix.}$$

$$C = \begin{bmatrix} C_1 & -C_1 \\ -C_1 & C_1 + C_2 \end{bmatrix} \text{ (c). Damping matrix.}$$

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \text{ (d). Position vector.}$$

$$\dot{Z} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} \text{ (e) Velocity vector.}$$

$$\ddot{Z} = \begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \end{bmatrix} \text{(f) Acceleration vector.}$$

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \text{(g) Force vector.}$$

The force balance equation can be used, or the Lagrange equation of the second type can be used. Either way, the damping coefficient  $c$  is very complicated to determine.

**Solution**

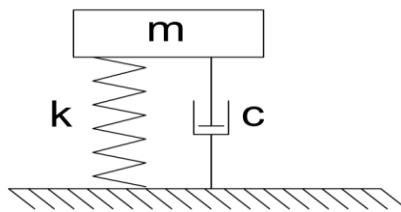


Figure 2. Model of one-degree-of-freedom harmonic oscillation ( $m$ : mass,  $c$ : damping coefficient,  $k$ : stiffness).

Consider a system that vibrates with one harmonic degree of freedom.

Equation of harmonic oscillation:

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{2}$$

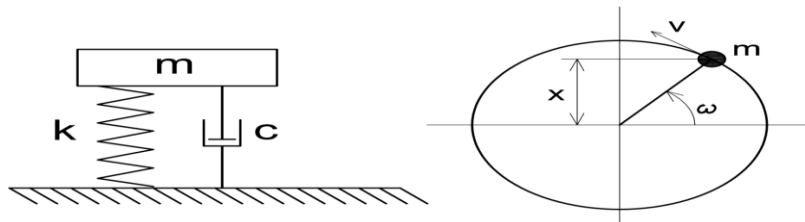


Figure 3. The equivalent physical model can describe this equation.

( $m$ : mass,  $c$ : damping coefficient,  $k$ : stiffness,  $x$ : distance,  $v$ : tangential velocity,  $\omega$ : angular

velocity).

In the free oscillation equation:

$$m\ddot{x} + kx = 0 \tag{3}$$

So, only parameter left:  $c\dot{x} = F_f$ , This is the power dissipation parameter.

Let the object m move on the distance S:

$$c\dot{x}.S = F_f.S \tag{4}$$

On the other hand:

$$F_f.S = \frac{1}{2}m.v^2 \tag{5}$$

While  $S \rightarrow 0$  then  $v \rightarrow 0$ .

The total amount of energy dissipated from when the object has velocity v until v = 0,

$$E = \frac{1}{2}m.v^2,$$

$$S = v.t \tag{6}$$

$$\dot{x} = v \tag{7}$$

Equation (5) can be rewritten as

$$c.v.v.t = \frac{1}{2}m.v^2 \tag{8}$$

So:

$$c.t = \frac{1}{2}m \Leftrightarrow c = \frac{m}{2t} (**)$$

Beside that

$$t = \frac{v}{a} = \frac{\omega.R}{a}; a = \frac{v^2}{R} = \frac{\omega^2.R^2}{R} \tag{9}$$

$$\Rightarrow t = \frac{\omega.R}{\frac{\omega^2.R^2}{R}} = \frac{1}{\omega} (***) \tag{10}$$

With  $\omega = \sqrt{\frac{k}{m}}$ ; Take the value of t for equation (10)

Obtained:

$$c = \frac{m}{2 \times \frac{1}{\omega}} = \frac{m}{2 \times \frac{1}{\sqrt{\frac{k}{m}}}} = \frac{1}{2} \sqrt{mk} \quad (11)$$

The final result shows that damping coefficient  $c$  depends only on two parameters -  $m$  and  $k$ , and  $c$  is not an independent parameter.

For a system of many degrees of freedom

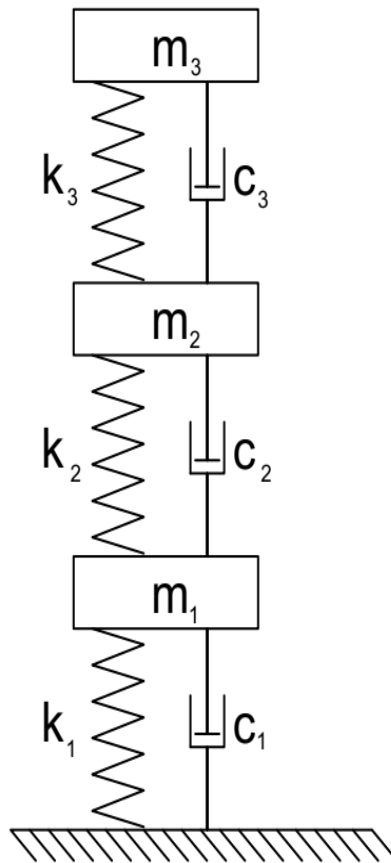


Figure 4. Example of multiple degrees of freedom model of harmonic oscillation.

Can be split into a degree of freedom to calculate the damping coefficient, then use the principle of additive effect to calculate.

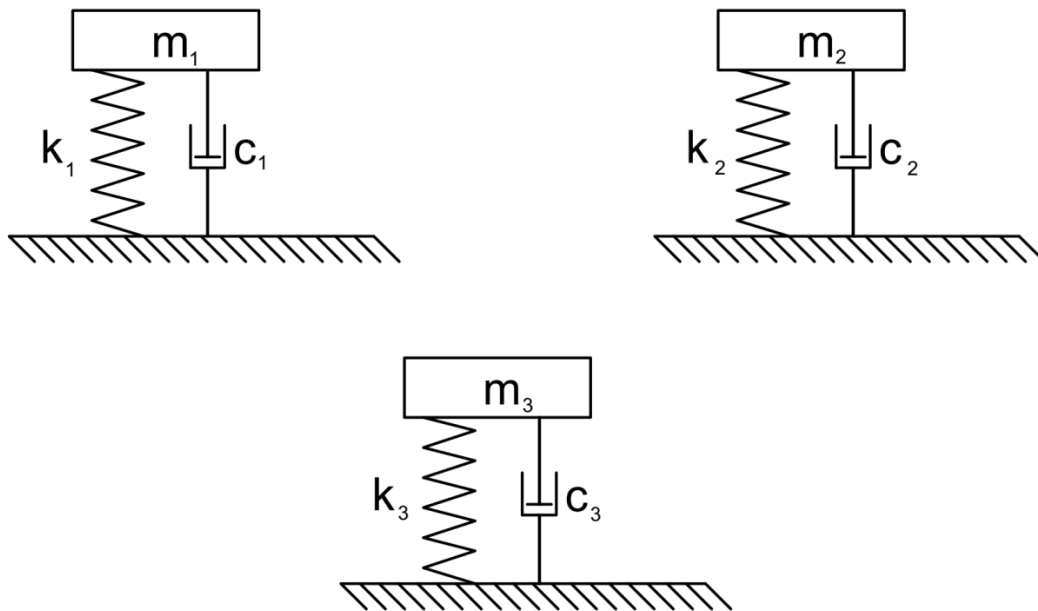


Figure5. The 3-degree-of-freedom model is divided into three independent single-degree-of-freedom models.

### Results and Discussions

$$c = \frac{1}{2} \sqrt{mk} \quad (12)$$

With: c: damping coefficient; m: mass; k: stiffness.

This is the formula for determining the damping coefficient.

Until now, the force balance equation, or Lagrange equation of type II, is still the primary mathematical basis for solving the problem of vibration mechanics.

The damping coefficient C depends on mass and stiffness. The fact that c is determined by the above formula fundamentally changes how the problem is solved. It helps to reduce the amount of computation significantly. In terms of physics, up to now, classical knowledge has considered the damping coefficient c as an independent parameter. However, the paper proved that it is a derived parameter.

### **Conclusion**

The most important implications of this study include two points. First, it provides a breakthrough for current computation in terms of mathematics and physics. Second, a series of follow-up studies were conducted to prove the correctness of the formula, making an essential contribution to understanding the physical nature.

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