Detection of the Edges of the Squamous Cell Carcinoma Using Fuzzy Partial Ro-transform

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Abstract
In both science and engineering, fuzzy partial differential equations are often used. The fuzzy Ro-transform transform and highly generalized H-differentiability notions are used in this study to propose a novel approach for solving fuzzy partial differential equations. The nth-order fuzzy partial derivative is given a general formula. By taking into account the borders of squamous cell carcinoma found through image processing after solving the wave equation and employing its answers, a squamous cell carcinoma application has been provided to illustrate the capabilities of the suggested approach.

Keywords: Partial Ro-Transform, Fuzzy Partial Ro-Transform, the Squamous Cell Carcinoma, Detection of the Edges of the Squamous Cell Carcinoma.

Introduction

It is required to use time-consuming, ineffective procedures to construct an accurate differential equation model for these issues since fuzzy differential equations have recently drawn a lot of attention. The fuzzy math technique is the approach that best deals with these issues. L. introduced the fundamental concept and mathematical foundation of fuzzy sets first. Zadeh, A., in [24]. Derivatives and fuzzy integration were studied by researchers [9,19], [7,9,1,14]. When regular differential equations are not always adequate for tackling such practical situations, a partial differential equation is utilized because it deals with several variables and how they are obtained.. This is because observing events frequently requires juggling multiple tasks at once; for example, while simulating the heat transmission of a wire, we must account for both time and distance. While fuzzy integrals can be used to solve linear partial differential equations, as those described in [1, 18], many writers have shown how to solve partial differential equations analytically and numerically [10,12]. Since Buckley and Feuring's initial suggestion of a partial differential equation for the fuzzy-valued function, there has been a ton of research in this area [15]. The handling of fuzzy partial differential equations is discussed differently by Allahviranloo in [2], and the approximate solution of the fuzzy heat equation using the Adomian decomposition method is examined in [4]. Chen et al. offered a unique inference approach with applications to FPDEs [8], and obergugoenberger offered weak and fuzzy solutions for these equations as well [16]. [6] discusses the analysis of wave, heat, and Poisson equations with unknown parameters. Bede et al. [5] originally introduced the fuzzy Fourier series, and as an application, they provided a cutting-edge method for fingerprint coding based on the extended Hukuhara partial derivative. Recently, Allahviranloo et al. [3] conducted research into the existence and uniqueness of the fuzzy heat equation solution and developed their analytical solutions.

In order to get the solution of the fuzzy wave equation using the fuzzy Ro-transform, this paper introduces the improper integral for a multivariate fuzzy-valued function and develops many requirements for both the fuzzy improper integral and extended Hukuhara partial differentiability. This article is divided into the following sections: results for the partition Ro-transform for fuzzy partial derivatives in the nth orders; and a general discussion of some basic ideas related to fuzzy numbers and fuzzy functions. It is feasible to resolve a wave equation using these formulas.

2. Basic concepts

Definition 1 [13]

In parametric form, a fuzzy number is a pair. $(\underline{\omega},\overline{\omega})$ of functions $\underline{\omega}(\vartheta),\overline{\omega}(\vartheta)$, $0\le\vartheta\le1$, which fulfills the following criteria:

1. $\underline{\omega}(\vartheta)$ is a left continuous function that is bounded and not decreasing in $(0,1]$,and right continuous at 0
2. $\overline{\omega}(\vartheta)$ is a left continuous function that is bounded and not rising. $(0,1]$,and right continuous at 0
3. $\underline{\omega}(\vartheta)\le\overline{\omega}(\vartheta)$, $0\le\vartheta\le1$.

Theorem 1. [22]

Let $\varsigma: R\to \mathrm{E}$ and it is portrayed by $\left[\underline{\varsigma}(\varpi;\vartheta),\overline{\varsigma}(\varpi;\vartheta)\right]$. For any fixed $\vartheta\in(0,1]$ propose that $\underline{\varsigma}(\varpi;\vartheta)$ and $\overline{\varsigma}(\varpi;\vartheta)$ that can be Riemann-integrated on [a, b] for every $b\ge a$, and assume There are two beneficial effects. $\underline{M}_\vartheta$ and $\overline{M}_\vartheta$ such that $\int_a^b\left|\underline{\varsigma}(\varpi;\vartheta)\right| d\varpi\le\underline{M}_\vartheta$ and $\int_a^b\left|\overline{\varsigma}(\varpi;\vartheta)\right| d\varpi\le\overline{M_\vartheta}$ for every $b\ge a$.Then, $\varsigma(\varpi)$ is the Riemann integrable fuzzy incorrect $[a,\infty)$ Additionally, we have: $\int_a^\infty\varsigma(\varpi)\ d\varpi=\left[\int_a^\infty\underline{\varsigma}(\varpi;\vartheta)\ d\varpi,\int_a^\infty\overline{\varsigma}(\varpi;\vartheta)\ d\varpi\right]$.

Definition2.[20]

Imagine that $\omega,\nu\in\mathrm{E}$ If $\exists\rho\in\mathrm{E}$ such that $\omega+\nu=\rho$ then ρ referred to as the Hukuhara difference. of ω and ν and it is identify by $\omega\ominus\nu$.

Definition 3. [21]

Let $u:(a,b)\times(a,b)\to\mathrm{E}$,is side to be nth order H-differentiable at $\varpi_0\in(a,b)$, with regard to x, if $\exists\frac{\partial^n}{\partial x^n}u(\varpi_0,\ddot{y})\in\mathrm{E}$ such that:

1. $\forall h > 0$ adequately small $\exists \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0 + h, ÿ) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0, ÿ), \exists \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0, ÿ) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0 - h, ÿ)$, then the following limit hold

$$\lim_{h \to 0^+} \frac{\frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0 + h, ÿ) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0, ÿ)}{h} = \lim_{h \to 0^+} \frac{\frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0, ÿ) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0 - h, ÿ)}{h} = \frac{\partial^n}{\partial \varpi^n} u(\varpi_0, ÿ)$$

Or

2. $\forall h > 0$ adequately small

$\exists \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0, ÿ) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0 + h, ÿ), \exists \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0 - h, ÿ) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0, ÿ)$, then the following limit hold

$$\lim_{h \to 0^+} \frac{\frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0, ÿ) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0 + h, ÿ)}{-h} = \lim_{h \to 0^+} \frac{\frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0 - h, ÿ) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi_0, ÿ)}{-h} \quad \frac{\partial^n}{\partial \varpi^n} u(\varpi_0, ÿ)$$

Definition 4. [21]

Let $u:(a,b)\times(a,b) \to \mathrm{E}$ (E: the set of all fuzzy numbers), is side to be H-differentiable of the nth order at $ÿ_0 \in (a,b)$, with respect to $ÿ$, if there exists an element $\frac{\partial^n}{\partial \varpi^n} u(\varpi, ÿ_0) \in \mathrm{E}$ such that:

1. $\forall h > 0$ sufficiently small

$\exists \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi, ÿ_0 + h) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi, ÿ_0), \exists \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi, ÿ_0) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi, ÿ_0 - h)$ then the following limit hold

$$\lim_{h \to 0^+} \frac{\frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi, ÿ_0 + h) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi, ÿ_0)}{h} = \lim_{h \to 0^+} \frac{\frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi, ÿ_0) \ominus \frac{\partial^{n-1}}{\partial \varpi^{n-1}} u(\varpi, ÿ_0 - h)}{h} = \frac{\partial^n}{\partial ÿ^n} u(\varpi, ÿ_0) .$$

Or

2. $\forall h>0$ sufficiently small

$\exists \frac{\partial^{n-1}}{\partial \ddot{y}^{n-1}}u(\varpi,\ddot{y}_0)\ominus\frac{\partial^{n-1}}{\partial \ddot{y}^{n-1}}u(\varpi,\ddot{y}_0+h),\exists\frac{\partial^{n-1}}{\partial \ddot{y}^{n-1}}u(\varpi,\ddot{y}_0-h)\ominus\frac{\partial^{n-1}}{\partial \ddot{y}^{n-1}}u(\varpi,\ddot{y}_0)$ then the following limit hold

$$\lim_{h\to 0^+}\frac{\frac{\partial^{n-1}}{\partial \ddot{y}^{n-1}}u(\varpi,\ddot{y}_0)\ominus\frac{\partial^{n-1}}{\partial t^{n-1}}u(\varpi,\ddot{y}_0+h)}{-h}=\lim_{h\to 0^+}\frac{\frac{\partial^{n-1}}{\partial t^{n-1}}u(\varpi,\ddot{y}_0-h)\ominus\frac{\partial^{n-1}}{\partial \ddot{y}^{n-1}}u(\varpi,\ddot{y}_0)}{-h}=\frac{\partial^{n}}{\partial \ddot{y}^{n}}u(\varpi,\ddot{y}_0)\ .$$

Definition 5.[17]

Assume that $u=u(\varpi,\ddot{y})$ be a fuzzy valued function and $\ddot{y}$ is real parameter, then fuzzy Ro-transform of the function u. Denote by $\Re_t(\varpi,\upsilon)$ is defined as follows:

$$\Re_{\ddot{y}}(\varpi,\upsilon)=R_{\ddot{y}}\left[u(\varpi,\ddot{y})\right]=\upsilon^2\int_0^{\infty}e^{-(i\sqrt[\Omega]{\upsilon})\ddot{y}}u(\varpi,\ddot{y})\,d\ddot{y}=\lim_{\tau\to\infty}\upsilon^2\int_0^{\tau}e^{-(i\sqrt[\Omega]{\upsilon})\ddot{y}}u(\varpi,t)\,d\ddot{y},$$

$$\Re_{\ddot{y}}(\varpi,\upsilon)=\left[\lim_{\tau\to\infty}\upsilon^2\int_0^{\tau}e^{-(i\sqrt[\Omega]{\upsilon})\ddot{y}}\underline{u}(\varpi,\ddot{y})\,d\ddot{y},\lim_{\tau\to\infty}\upsilon^2\int_0^{\tau}e^{-(i\sqrt[\Omega]{\upsilon})\ddot{y}}\overline{u}(\varpi,\ddot{y})\,d\ddot{y}\right].$$

Whenever the limits exist. The $\vartheta-$cut representation of $\Re_{\ddot{y}}(\varpi,\upsilon)$ is given as:

$$\Re_{\ddot{y}}(\varpi,\upsilon;\vartheta)=R_{\ddot{y}}\left[u(\varpi,\ddot{y};\vartheta)\right]=\left[\gamma(\underline{u}(\varpi,\ddot{y})),\gamma(\overline{u}(\varpi,\ddot{y}))\right],$$

Theorem 4.[11] Let $u:(0,\infty)\times(0,\infty)\to E$ be a function and denote by $\left[u(\varpi,\ddot{y})\right]^{\vartheta}=\left[\underline{u}(\varpi,\ddot{y};\vartheta),\overline{u}(\varpi,\ddot{y};\vartheta)\right]$. Such that $\underline{u}(\varpi,\ddot{y};\vartheta)$ and $\overline{u}(\varpi,\ddot{y};\vartheta)$ are differentiable functions of the n[th] order with respect to $\ddot{y}$, then:

1. If $u(\varpi,\ddot{y})$ is the first form differentiable function, then:
$$\left[\frac{\partial^n}{\partial \ddot{y}^n}u(\varpi,\ddot{y})\right]^{\vartheta}=\left[\frac{\partial^n}{\partial \ddot{y}^n}\underline{u}(\varpi,\ddot{y};\vartheta),\frac{\partial^n}{\partial \ddot{y}^n}\overline{u}(\varpi,\ddot{y};\vartheta)\right],$$
2. If $u(\varpi,\ddot{y})$ is the second form differentiable function, then :
$$\left[\frac{\partial^n}{\partial \ddot{y}^n}u(\varpi,\ddot{y})\right]^{\vartheta}=\left[\frac{\partial^n}{\partial \ddot{y}^n}\overline{u}(\varpi,\ddot{y};\vartheta),\frac{\partial^n}{\partial \ddot{y}^n}\underline{u}(\varpi,\ddot{y};\vartheta)\right].$$

3. Fuzzy Partial Derivative Formulas for nth Orders.

Theorem 7: Let $u:(0,\infty)\times(0,\infty)\to E$ be a function and $u(\varpi,\ddot{y}),\frac{\partial}{\partial \ddot{y}}u(\varpi,\ddot{y}),\ldots,\frac{\partial^{n-1}}{\partial \ddot{y}^{n-1}}u(\varpi,\ddot{y})$ are continuous fuzzy-valued functions and of exponential order and $\frac{\partial^n}{\partial \ddot{y}^n}u(\varpi,\ddot{y})$ is piecewise continuous fuzzy-valued functions.

Let $\frac{\partial^{j_1}}{\partial \ddot{y}^{j_1}} u(\varpi,\ddot{y}), \frac{\partial^{j_2}}{\partial \ddot{y}^{j_2}} u(\varpi,\ddot{y}),\ldots,\frac{\partial^{j_\mu}}{\partial \ddot{y}^{j_\mu}} u(\varpi,\ddot{y})$ are the second form differentiable functions for $0 \le j_1 < j_2 < \ldots < j_\mu \le n-1, 0 \le \mu \le n$ and $\frac{\partial^q}{\partial t^q} u(\varpi,\ddot{y})$ is the first form differentiable function for $q \ne j_\tau, \tau = 1,2,\ldots,\mu$, and if ϑ-cut representation of fuzzy-valued function is denote by:

(1)If μ is an even number, then:

$$R_{\ddot{y}}\left(\frac{\partial^n}{\partial \ddot{y}^n} u(\varpi,\ddot{y})\right) = \left(i\sqrt[\Omega]{\upsilon}\right)^n R_{\ddot{y}}\left(u(\varpi,\ddot{y})\right) \ominus \upsilon^2 \left(i\sqrt[\Omega]{\upsilon}\right)^{n-1} u(\varpi,0) ® \upsilon^2 \sum_{\kappa=1}^{n-1} \left(i\sqrt[\Omega]{\upsilon}\right)^{n-(k+1)} \frac{\partial^k}{\partial \ddot{y}^k} u(\varpi,0).$$

such that

$$® = \begin{cases} \ominus, & \text{if the number of the second form differentiable functions between } j_1,\ldots,j_k \\ & \text{is an even number.} \\ -, & \text{if the number of the second form differentiable functions between } j_1,\ldots,j_k \\ & \text{is an odd number.} \end{cases} \tag{5}$$

2. If μ is an odd number, then:

$$R_{\ddot{y}}\left(\frac{\partial^n}{\partial \ddot{y}^n} u(\varpi,\ddot{y})\right) = -\upsilon^2 \left(i\sqrt[\Omega]{\upsilon}\right)^{n-1} u(\varpi,0) \ominus \left(-i\sqrt[\Omega]{\upsilon}\right)^n R_{\ddot{y}}\left(u(\varpi,\ddot{y})\right) ® \upsilon^2 \sum_{\kappa=1}^{n-1} \left(i\sqrt[\Omega]{\upsilon}\right)^{n-(k+1)} \frac{\partial^k}{\partial \ddot{y}^k} u(\varpi,0).$$

such that

$$® = \begin{cases} \ominus, & \text{if the number of second form differentiable functions between } j_1,\ldots,j_k \\ & \text{is an odd number.} \\ -, & \text{if the number of second form differentiable functions between } j_1,\ldots,j_k \\ & \text{is an even number.} \end{cases} \tag{6}$$

Proof1: **If μ is an even number**

Since $\frac{\partial^{j_1}}{\partial \ddot{y}^{j_1}} u(\varpi,\ddot{y}), \frac{\partial^{j_2}}{\partial \ddot{y}^{\partial^{j_2}}} u(\varpi,\ddot{y}),\ldots,\frac{\partial^{j_\mu}}{\partial \ddot{y}^{j_\mu}} u(\varpi,\ddot{y})$ are the second form differentiable functions and μ is an even number, By Theorem 4/1:

$$\frac{\partial^n}{\partial \ddot{y}^n} u(\varpi,\ddot{y}) = \left(\frac{\partial^n}{\partial \ddot{y}^n} \underline{u}(\varpi,\ddot{y};\vartheta), \frac{\partial^n}{\partial \ddot{y}^n} \overline{u}(\varpi,\ddot{y};\vartheta)\right).$$

$$= \left(\gamma_{\ddot{y}}\left[\frac{\partial^n}{\partial \ddot{y}^n} \underline{u}(\varpi,\ddot{y};\vartheta)\right], \gamma_{\ddot{y}}\left[\frac{\partial^n}{\partial \ddot{y}^n} \overline{u}(\varpi,\ddot{y};\vartheta)\right]\right). \tag{7}$$

Then:

$$\begin{aligned}\gamma_{\ddot{y}}\left[\frac{\partial^n}{\partial \ddot{y}^n}\underline{u}(\varpi,\ddot{y};\vartheta)\right]&=\left(i\sqrt[\Omega]{\upsilon}\right)^n\gamma_{\ddot{y}}\left[\underline{u}(\varpi,\ddot{y};\vartheta)\right]-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)^{n-1}\underline{u}(\varpi,0;\vartheta)\\&-\upsilon^2\sum_{k=1}^{k_1-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-k-1}\frac{\partial^k}{\partial \ddot{y}^k}\underline{u}(\varpi,0;\vartheta)-\upsilon^2\sum_{k=k_1}^{k_2-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-k-1}\frac{\partial^k}{\partial \ddot{y}^k}\underline{u}(\varpi,0;\vartheta)\\&-\ldots-\upsilon^2\sum_{k=k_{\mu-1}}^{k_\mu-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-k-1}\frac{\partial^k}{\partial \ddot{y}^k}\underline{u}(\varpi,0;\vartheta)-\upsilon^2\sum_{k=k_\mu}^{n-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-k-1}\frac{\partial^k}{\partial \ddot{y}^k}\underline{u}(\varpi,0;\vartheta).\end{aligned}\tag{8}$$

In similar way can be get:

$$\begin{aligned}\gamma_{\ddot{y}}\left[\frac{\partial^n}{\partial \ddot{y}^n}\bar{u}(\varpi,\ddot{y};\vartheta)\right]&=\left(i\sqrt[\Omega]{\upsilon}\right)^n\gamma_{\ddot{y}}\left[\underline{u}(\varpi,\ddot{y};\vartheta)\right]-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)^{n-1}\bar{u}(\varpi,0;\vartheta)\\&-\upsilon^2\sum_{k=1}^{k_1-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-k-1}\frac{\partial^k}{\partial \ddot{y}^k}\bar{u}(\varpi,0;\vartheta)-\upsilon^2\sum_{k=k_1}^{k_2-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-k-1}\frac{\partial^k}{\partial \ddot{y}^k}\bar{u}(\varpi,0;\vartheta)\\&-\ldots-\upsilon^2\sum_{k=k_{\mu-1}}^{k_\mu-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-k-1}\frac{\partial^k}{\partial \ddot{y}^k}\bar{u}(\varpi,0;\vartheta)-\upsilon^2\sum_{k=k_\mu}^{n-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-k-1}\frac{\partial^k}{\partial \ddot{y}^k}\bar{u}(\varpi,0;\vartheta).\end{aligned}\tag{9}$$

Since $j_{k_1}=j_{k_2}=\ldots=j_{k_\mu}=$ are the second differentiable functions, when μ is an even number then have been the following relation:

$$\begin{aligned}&\frac{\partial^k}{\partial \ddot{y}^k}\underline{u}(\varpi,0;\vartheta)=\frac{\partial^k}{\partial \ddot{y}^k}\underline{u(\varpi,0;\vartheta)},\frac{\partial^k}{\partial \ddot{y}^k}\bar{u}(\varpi,0;\vartheta)=\frac{\partial^k}{\partial \ddot{y}^k}\overline{u(\varpi,0;\vartheta)};1\le k\le k-1,\\&\frac{\partial^k}{\partial \ddot{y}^k}\bar{u}(\varpi,0;\vartheta)=\frac{\partial^k}{\partial \ddot{y}^k}\underline{u(\varpi,0;\vartheta)},\frac{\partial^k}{\partial \ddot{y}^k}\underline{u}(\varpi,0;\vartheta)=\frac{\partial^k}{\partial \ddot{y}^k}\overline{u(\varpi,0;\vartheta)};k_1\le k\le k_2-1,\\&\mathrm{M}\\&\frac{\partial^k}{\partial \ddot{y}^k}\bar{u}(\varpi,0;\vartheta)=\frac{\partial^k}{\partial \ddot{y}^k}\underline{u(\varpi,0;\vartheta)},\frac{\partial^k}{\partial \ddot{y}^k}\underline{u}(\varpi,0;\vartheta)=\frac{\partial^k}{\partial \ddot{y}^k}\overline{u(\varpi,0;\vartheta)};k_{\mu-1}\le k\le k_\mu-1,\\&\frac{\partial^k}{\partial \ddot{y}^k}\underline{u}(\varpi,0;\vartheta)=\frac{\partial^k}{\partial \ddot{y}^k}\underline{u(\varpi,0;\vartheta)},\frac{\partial^k}{\partial \ddot{y}^k}\bar{u}(\varpi,0;\vartheta)=\frac{\partial^k}{\partial \ddot{y}^k}\overline{u(\varpi,0;\vartheta)};k_\mu\le k\le n-1.\end{aligned}\tag{10}$$

The last one of the equations in (10) yields.

Because μ is an even number. Using (8), (9) and (10), equation (7) becomes

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$$R_{ÿ}\left(\frac{\partial^n}{\partial ÿ^n}u(\varpi,t)\right)=\left(i\sqrt[\Omega]{\upsilon}\right)^n R_{ÿ}\left(u(\varpi,ÿ)\right)\ominus\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)^{n-1}u(\varpi,0)\circledR\upsilon^2\sum_{\kappa=1}^{n-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-(k+1)}\frac{\partial^k}{\partial ÿ^k}u(\varpi,0).$$

Proof 2: **Since μ is an odd number**

By same way we can get:

$$R_{ÿ}\left(\frac{\partial^n}{\partial ÿ^n}u(\varpi,ÿ)\right)=-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)^{n-1}u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^n R_{ÿ}\left(u(\varpi,ÿ)\right)\circledR\upsilon^2\sum_{\kappa=1}^{n-1}\left(i\sqrt[\Omega]{\upsilon}\right)^{n-(k+1)}\frac{\partial^k}{\partial ÿ^k}u(\varpi,0).$$

4. Solve Fuzzy Wave Equation by Use Fuzzy Ro-transform

To solve fuzzy Wave Equation about second order by use Ro-transform, we have:

CaseA$_1$: Let's think about u is the first form differentiable functions or u is the second form differentiable functions, then we get the following:

$$R_{ÿ}\left(\frac{\partial^2 u(\varpi,ÿ)}{\partial\varpi^2}\right)=\left[\frac{\partial^2\underline{u}(\varpi,ÿ;\vartheta)}{\partial\varpi^2},\frac{\partial^2\overline{u}(\varpi,ÿ;\vartheta)}{\partial\varpi^2}\right],\ 0\le\vartheta\le1.$$

CaseA$_2$**:** Let's think about u is the second form differentiable function , then we get the following:

$$R_{ÿ}\left(\frac{\partial^2 u(\varpi,ÿ)}{\partial\varpi^2}\right)=\left[\frac{\partial^2\overline{u}(\varpi,ÿ;\vartheta)}{\partial\varpi^2},\frac{\partial^2\underline{u}(\varpi,ÿ;\vartheta)}{\partial\varpi^2}\right],\ 0\le\vartheta\le1.$$

Case1: Let's think about $u,u_{ÿ}$ is the first form differentiable functions, then we get the following:

$$R_{ÿ}\left[u_{ÿÿ}(\varpi,t)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2 R_{ÿ}\left[u(\varpi,ÿ)\right]\ominus\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\upsilon^2u_{ÿ}(\varpi,0)$$

Case2: Let's think about u is the first form differentiable function and $u_{ÿ}$ is the second form differentiable function, then we get the following:

$$R_{ÿ}\left[u_{ÿÿ}(\varpi,t)\right]=-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^2 R_{ÿ}\left[u(\varpi,ÿ)\right]\ominus\upsilon^2u_{ÿ}(\varpi,0).$$

Case3: Let's think about u is the second form differentiable function and $u_{ÿ}$ is the first form differentiable function, then we get the following:

$$R_{ÿ}\left[u_{ÿÿ}(\varpi,ÿ)\right]=-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^2 R_{ÿ}\left[u(\varpi,ÿ)\right]-\upsilon^2u_{ÿ}(\varpi,0).$$

Case4: Let's think about $u, u_{ÿ}$ is the second form differentiable functions, then we get the following:

$$R_{ÿ}\left[u_{ÿÿ}(\varpi,ÿ)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2 R_{ÿ}\left[u(\varpi,ÿ)\right]\ominus\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)-\upsilon^2 u_{ÿ}(\varpi,0).$$

Example 1: Consider the following fuzzy partial differential equation:

$u_{\varpi\varpi}=u_{ÿÿ}$; I.C. (i) $u(\varpi,0)=u_{ÿ}(\varpi,0)=(2\vartheta-1,2-\vartheta)$,

B.C. (ii) $u(0,ÿ)=(\vartheta-1,1-\vartheta)$, $\lim\limits_{\varpi\to\infty}$ exist.

Ro-transform is used to solve the initial problem on both sides. $R_{ÿ}\left[u_{\varpi\varpi}(\varpi,ÿ)\right]=R_{ÿ}\left[u_{tt}(\varpi,ÿ)\right]$.

We get the following:

Cases (A_1&1) or (A_2&3).

$$\frac{\partial^2}{\partial\varpi^2}R_{ÿ}\left[u(\varpi,ÿ)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2 R_{ÿ}\left[u(\varpi,ÿ)\right]\ominus\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\upsilon^2 u_{ÿ}(\varpi,0).$$

Using initial conditions:

$$\frac{\partial^2}{\partial\varpi^2}\gamma_{ÿ}\left[\underline{u}(\varpi,ÿ;\vartheta)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2\gamma_{ÿ}\left[\underline{u}(\varpi,ÿ;\vartheta)\right]-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)(2\vartheta-1)-\upsilon^2(2\vartheta-1),$$

$$\frac{\partial^2}{\partial\varpi^2}\gamma_{ÿ}\left[\bar{u}(\varpi,ÿ;\vartheta)\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2\gamma_{ÿ}\left[\bar{u}(\varpi,ÿ;\vartheta)\right]-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)(2-\vartheta)-\upsilon^2(2-\vartheta).$$

Solve above equations, and using boundary conditions, then:

$$\gamma_{ÿ}\left[\underline{u}(\varpi,ÿ;\vartheta)\right]=\left[\frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}}(\vartheta-1)-\frac{\upsilon^2}{2i\sqrt[\Omega]{\upsilon}}(2\vartheta-1)-\frac{\upsilon^2}{2\left(i\sqrt[\Omega]{\upsilon}\right)^2}(2\vartheta-1)\right]e^{-i\sqrt[\Omega]{\upsilon}\varpi}$$
$$-\frac{\upsilon^2}{2i\sqrt[\Omega]{\upsilon}}(2\vartheta-1)-\frac{\upsilon^2}{2\left(i\sqrt[\Omega]{\upsilon}\right)^2}(2\vartheta-1),$$

$$\gamma_{ÿ}\left[\bar{u}(\varpi,ÿ;\vartheta)\right]=\left[\frac{\upsilon^2}{i\sqrt[\Omega]{\upsilon}}(1-\vartheta)-\frac{\upsilon^2}{2i\sqrt[\Omega]{\upsilon}}(2-\vartheta)-\frac{\upsilon^2}{2\left(i\sqrt[\Omega]{\upsilon}\right)^2}(2-\vartheta)\right]e^{-i\sqrt[\Omega]{\upsilon}\varpi}$$
$$-\frac{\upsilon^2}{2i\sqrt[\Omega]{\upsilon}}(2-\vartheta)-\frac{\upsilon^2}{2\left(i\sqrt[\Omega]{\upsilon}\right)^2}(2-\vartheta).$$

By applying the fuzzy Ro-inverse transform to both sides of the equation, we finally obtain the solutions as follows.

$$\underline{u}(\varpi,\ddot{y};\vartheta)=\left[(\vartheta+1)-\frac{1}{2}(2\vartheta-1)-\frac{\ddot{y}}{2}(2\vartheta-1)\right]H(\ddot{y}-\varpi)-\frac{1}{2}(2\vartheta-1)-\frac{\ddot{y}}{2}(2\vartheta-1),$$

$$\bar{u}(\varpi,\ddot{y};\vartheta)=\left[(1-\vartheta)-\frac{1}{2}(2-\vartheta)-\frac{\ddot{y}}{2}(2-\vartheta)\right]H(\ddot{y}-\varpi)-\frac{1}{2}(2-\vartheta)-\frac{\ddot{y}}{2}(2-\vartheta).$$

Cases (A_1&2) or (A_2&4).

$$\frac{\partial^2}{\partial\varpi^2}R_{\ddot{y}}\left[u(\varpi,\ddot{y})\right]=-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^2R_{\ddot{y}}\left[u(\varpi,\ddot{y})\right]\ominus\upsilon^2u_{\ddot{y}}(\varpi,0).$$

Then:

$$\underline{u}(\varpi,\ddot{y};\vartheta)=\left[(\vartheta+1)-\frac{1}{2}(3\vartheta-3)\right]H(\ddot{y}-\varpi)-\frac{1}{2}(3\vartheta-3),$$

$$\bar{u}(\varpi,\ddot{y};\vartheta)=\left[(1-\vartheta)-\frac{1}{2}(3-3\vartheta)\right]H(\ddot{y}-\varpi)-\frac{1}{2}(3-3\vartheta).$$

Cases (A_1&3) or (A_2&1).

$$\frac{\partial^2}{\partial\varpi^2}R_{\ddot{y}}\left[u(\varpi,\ddot{y})\right]=-\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)\ominus\left(-i\sqrt[\Omega]{\upsilon}\right)^2R_{\ddot{y}}\left[u(\varpi,\ddot{y})\right]-\upsilon^2u_{\ddot{y}}(\varpi,0).$$

Then:

$$\underline{u}(\varpi,\ddot{y};\vartheta)=-\frac{1}{2}(2\vartheta-1)-\frac{\ddot{y}}{2}(2\vartheta-1),$$

$$\bar{u}(\varpi,\ddot{y};\vartheta)=-\frac{1}{2}(2-\vartheta)-\frac{\ddot{y}}{2}(2-\vartheta).$$

Cases (A_1&4) or (A_2&2).

$$\frac{\partial^2}{\partial\varpi^2}R_{\ddot{y}}\left[u(\varpi,\ddot{y})\right]=\left(i\sqrt[\Omega]{\upsilon}\right)^2R_{\ddot{y}}\left[u(\varpi,\ddot{y})\right]\ominus\upsilon^2\left(i\sqrt[\Omega]{\upsilon}\right)u(\varpi,0)-\upsilon^2u_{\ddot{y}}(\varpi,0).$$

Then:

$$\underline{u}(\varpi,\ddot{y};\vartheta)=\left[(\vartheta+1)-\frac{1}{2}(2\vartheta-1)-\frac{\ddot{y}}{2}(2-\vartheta)\right]H(\ddot{y}-\varpi)-\frac{1}{2}(2\vartheta-1)-\frac{\ddot{y}}{2}(2-\vartheta),$$

$$\bar{u}(\varpi,\ddot{y};\vartheta)=\left[(1-\vartheta)-\frac{1}{2}(2-\vartheta)-\frac{\ddot{y}}{2}(2\vartheta-1)\right]H(\ddot{y}-\varpi)-\frac{1}{2}(2-\vartheta)-\frac{\ddot{y}}{2}(2\vartheta-1).$$

4. Squamous Cell Carcinoma

Squamous cell carcinoma (SCC), the second most common kind of skin cancer, is defined by abnormal, accelerated squamous cell proliferation. If found early, the majority of SCCs are curable.

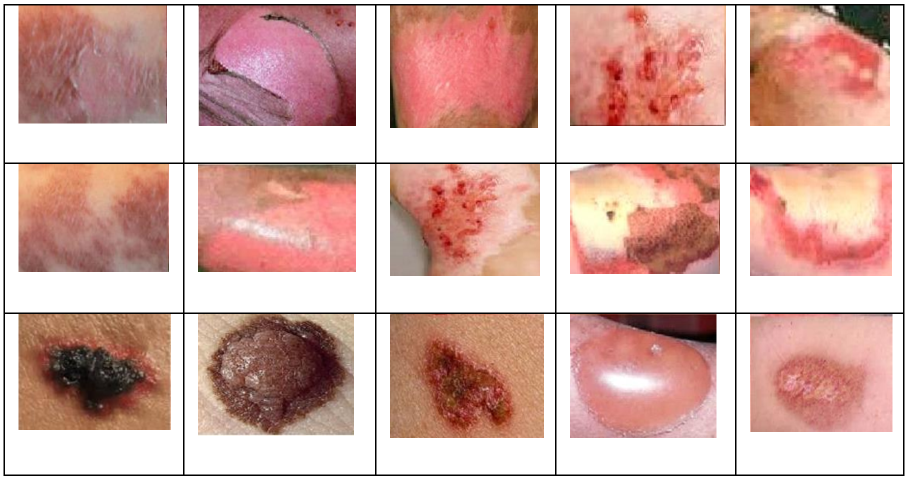


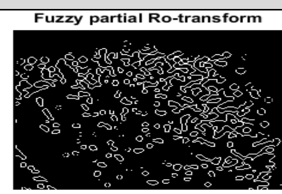
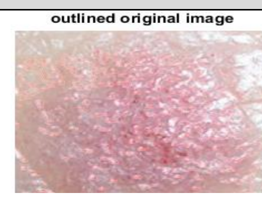
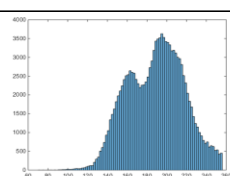
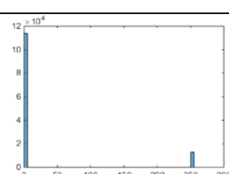
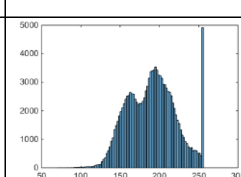
Table (1)

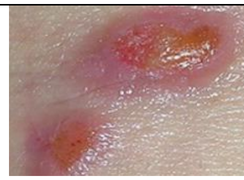
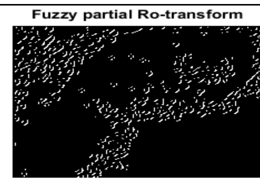
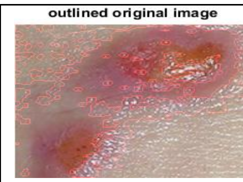
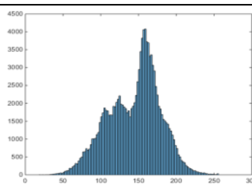
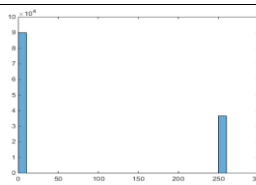
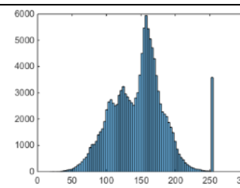
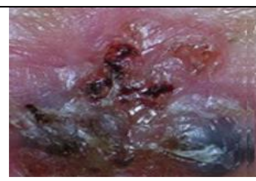
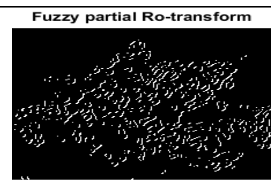
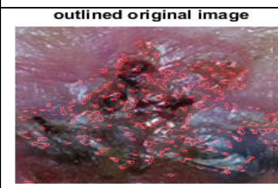
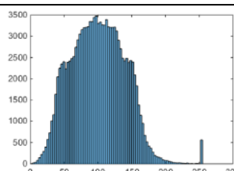
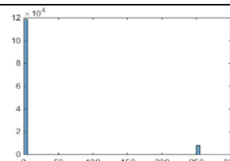
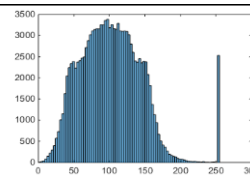
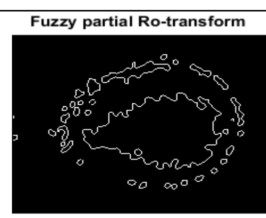
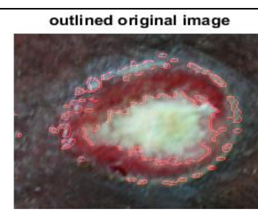
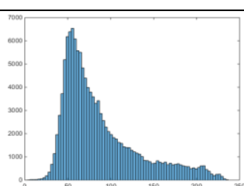
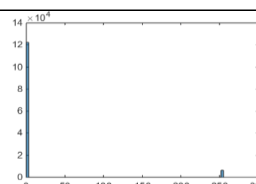
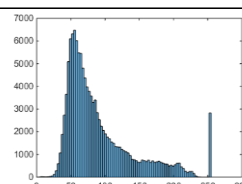
5. Determine the edge disease of Squamous Cell Carcinoma using image processing

A new picture g is often defined by a previous image f in an image processing activity. The easiest operations are those that change each individual pixel.

6. Application

In this research, we will use wave equation solutions obtained from image processing to extract the margins of the cancer disease.

Cases	Orange Image	Fuzzy Partial Ro – transform	Out lined original Image
Case1			
Histogram			
Mean	188.5465	25.9336	190.0671
STD	28.3416	77.0751	30.5077

Case2			
Histogram			
Mean	**144.1336**	**13.3896**	**145.7143**
STD	**34.4746**	**56.8778**	**37.3790**
Case3			
Histogram			
Mean	**99.9143**	**16.0409**	**102.8064**
STD	**39.6118**	**61.9125**	**44.9740**
Case4			
Histogram			
Mean	**90.0524**	**12.5589**	**90.8136**

STD	**44.3997**	**55.1799**	**50.3335**

Table (2)

Conclusion

In this work, general-form fuzzy derivatives of fuzzy Ro-transform generalized order were found, and wave equations were solved using these formulas. We employed wave equation solutions in image processing to find edges.

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