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Errors in Bernoulli's Law and How to Eliminate and Solve Them

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Abstract

Bernoulli's law is widely used in practice and the study of fluid mechanics. This law has existed since 1738 when *the Hydrodynamica book was published.* Until now, it has not been repaired. The base of the law has some inaccuracies that lead to mistakes in its application. Therefore, their repair is essential. This article uses theories of physics and mathematical logic to point out the law's flaws. Current knowledge determines that the law of conservation of energy is correct. This correctness is the main basis for pointing out the problems in Bernoulli's law. The form of approach and problem-solving is to consider exercises and classic examples in textbooks on fluid mechanics. After that, they are going to be compared with the right knowledge. From it, the truths are reached.

The study's main results pointed out Bernoulli's fundamental mistake, from applying basic knowledge to formulating the law. The errors of practical applications based on this knowledge are also clarified. From these results, there will be fundamental changes when building the theory for other sites. A series of applications in real life also change deeply. At the same time, they are leading to saving money, time, labor, and labor results and opening new doors of knowledge and science.

Keywords: Physics, math, Bernoulli’s law, errors, solving.

1. Introduction

Bernoulli's law is very familiar and has long been recognized by the scientific community as correct and widely applied. However, this does not mean that this law reflects the physical nature of the object of study. That object is an incompressible flow for a liquid. It is also possible to be a compressible flow for gases.

The fundamental problem to this day is that the law still was understood to be correct because the empirical coefficients have corrected the result from the wrong law calculation

These coefficients are difficult to verify correctness. After being miss-corrected, the results obtained are close to those that reflect nature. On the other hand, although there are errors in the calculation process applied, the equation is sometimes right and sometimes wrong. So, this has contributed to making mistakes hard to spot.

The equation was built on a fundamental error in terms of mathematical and physical logic.

The basic objective of this study is to solve these existing problems. Contribute to the creation of new physical bases for further research and applications.

2. Method

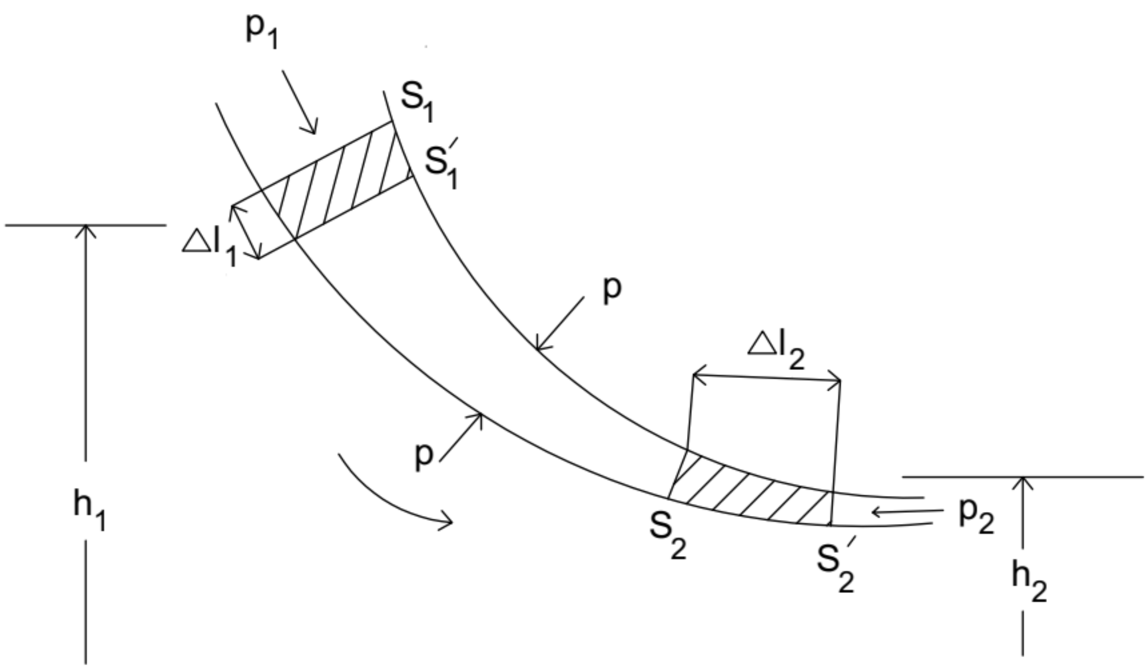

Figure 1. Diagram depicting how to establish the Bernoulli equation

We take in an ideal fluid in stationary motion, a stream tube with a small cross-section limited by s1 and s2, placed in the uniform gravity of the earth. After some time, Δt, the fluid moves, and s1 and s2 move to s1' and s2'. Due to the law of conservation of current

$$s_1.\Delta l_1 = s_2.\Delta l_2 \quad (1)$$

Since the fluid is ideal (no viscous forces), the work done by the pressure is exactly equal to the change in mechanical energy of the fluid mass (s1, s2) in motion

$$A = \Delta W \quad eq(2)$$

We take the flow pipe to be small enough; that is, s and l are small enough that all the points of each slash have the same velocity v, pressure p, and height h. Therefore the work of the pressure on the fluid mass (s1, s2) will be

$$A = p_1.s_1.\Delta l_1 - p_2.s_2.\Delta l_2 \quad (3)$$

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At the same time, the change in mechanical energy of the fluid in motion:

$$\Delta \boldsymbol{W} = \boldsymbol{W}(\boldsymbol{s}_1', \boldsymbol{s}_2') - (\boldsymbol{s}_1, \boldsymbol{s}_2) \qquad (4)$$

But (s_1, s_2) and (s_1', s_2') have the common part of (s_1', s_2) the unslashed part, so it can be

rewritten

$$\Delta \boldsymbol{W} = \boldsymbol{W}(\boldsymbol{s}_2, \boldsymbol{s}_2') - (\boldsymbol{s}_1, \boldsymbol{s}_1') \qquad (5)$$

Where the mechanical energy of each stored mass is equal to the sum of their kinetic and potential energies in uniform gravity, thus:

$$\Delta \boldsymbol{W} = \left(\frac{\boldsymbol{\rho s_2 \Delta l_2 v_2^2}}{\mathbf{2}} + \boldsymbol{\rho s_2 \Delta l_2 g h_2}\right) - \left(\frac{\boldsymbol{\rho s_1 \Delta l_1 v_1^2}}{\mathbf{2}} + \boldsymbol{\rho s_1 \Delta l_1 g h_1}\right) \quad (6)$$

Put (3), (6) in (2) and notice: $s_1 . \Delta l_1 = s_2 . \Delta l_2$ we can draw:

$$\frac{\boldsymbol{\rho v_1^2}}{\mathbf{2}} + \boldsymbol{\rho g h_1} + \boldsymbol{p_1} = \frac{\boldsymbol{\rho v_2^2}}{\mathbf{2}} + \boldsymbol{\rho g h_2} + \boldsymbol{p_2} \quad (7)$$

It is the equation of Bernoulli's law, which is the fundamental law of ideal fluid dynamics in uniform gravity. The law states: "In an ideal fluid in stationary motion, along each line, the flow always satisfies the condition.

$$\frac{\boldsymbol{\rho v^2}}{\mathbf{2}} + \boldsymbol{\rho g h} + \boldsymbol{p} = \boldsymbol{const} \quad (8)$$

The above formula has been believed to hold for a long time because when applied to real fluids with small viscous forces, it has a fairly good accuracy compared to reality.

In all the above logical arguments, there is a crucial mistake that is:

Eq(3) is misleading.

$$A = p_1 . s_1 . \Delta l_1 - p_2 . s_2 . \Delta l_2$$

$$A = p_1.s_1.\Delta l_1 - p_2.s_2.\Delta l_2 = p_1.s_1.v_1.t_1 - p_2.s_2.v_2.t_2$$

For simplicity, consider the following model.

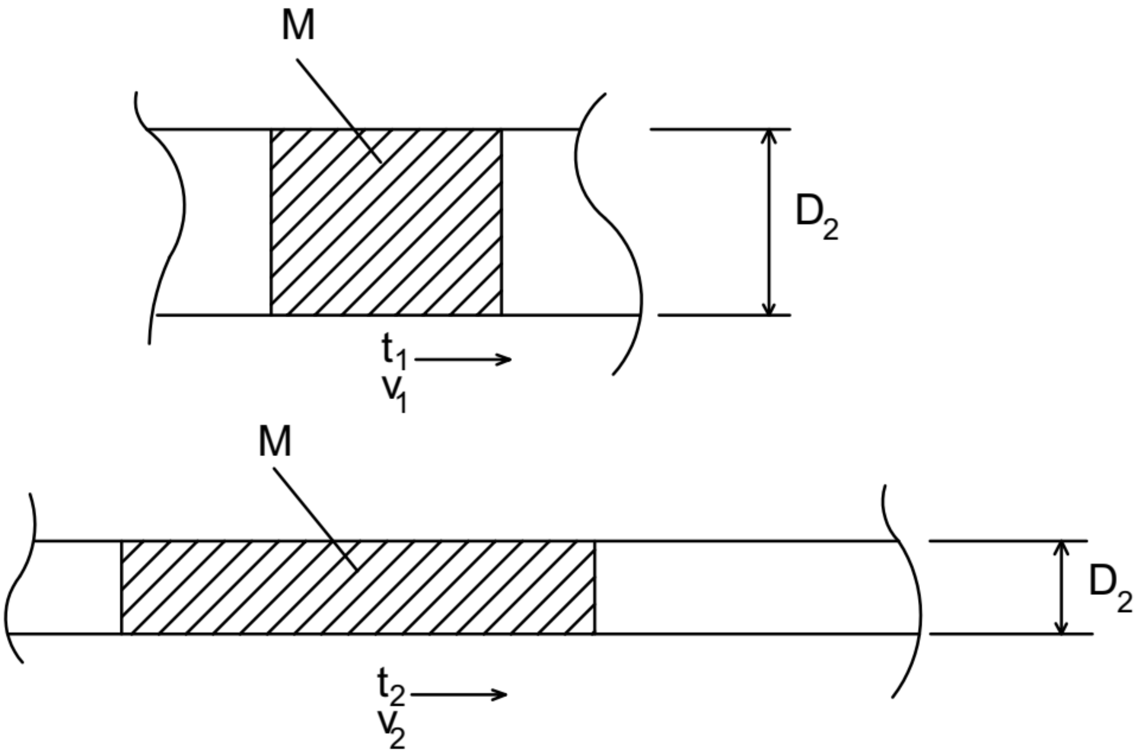

Figure. 2 The model describes the fluid mass M, moving in tubes of different diameters.

Volume M takes time t_1 to flow in a pipe section with diameter D_1

Also, the volume M, it will take time t_2 to soak in the pipe section with diameter D_2

It's easy to see $t_1 \neq t_2$

$$s_1.\Delta l_1 = s_2.\Delta l_2 \Rightarrow s_1.v_1.t_1 = s_2.v_2.t_2 \Rightarrow s_1.v_1 \neq s_2.v_2$$

Therefore, it can be concluded that Equation 3 did not consider the role of the time factor. Meanwhile, ΔW is calculated as follows.

$$\Delta W = \frac{1}{2}m_1v_1^2 + m_1gh_1 - \left(\frac{1}{2}m_2v_1^2 + m_2gh_2\right)$$

$$= \left(\frac{\rho s_2 v_2 t_2 v_2^2}{2} + \rho s_2 v_2 t_2 g h_2\right) - \left(\frac{\rho s_1 v_1 t_1 v_1^2}{2} + \rho s_1 v_1 t_1 g h_1\right)$$

$$= \left(\frac{\rho s_2 t_2 v_2^3}{2} + \rho s_2 v_2 t_2 g h_2\right) - \left(\frac{\rho s_1 t_1 v_1^3}{2} + \rho s_1 v_1 t_1 g h_1\right)$$

ΔW is calculated based on the assumption: In the same period t, the mass m at the first and second positions is the same ($m_1 = m_2$). However, if we consider the above model, if at the same time t, $D_1 \neq D_2$, then $m_1 \neq m_2$. This is the second basic mistake when calculatingΔW.

Eq(2) must be corrected:

A = ΔW.

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$$p_1.s_1.v_1.t_1 - p_2.s_2.v_2.t_2 = \left(\frac{\rho s_2 v_2 t_2 v_2^2}{2} + \rho s_2 v_2 t_2 g h_2\right) - \left(\frac{\rho s_1 v_1 t_1 v_1^2}{2} + \rho s_1 v_1 t_1 g h_1\right)$$

Because $s_1.v_1 \neq s_2.v_2$, it cannot be reduced to eq(8).

Based on this result, it can be confirmed that eq(7) and eq(8) are incorrect equations. Thus, all the consequences of Bernoulli's law are physically wrong.

Some examples of incorrect consequences are the following equations:

Bernoulli equation (BE), inviscid fluid: $\int \frac{\mathrm{d}p}{\gamma} + \frac{V^2}{2g} + z = C; \int \frac{\mathrm{d}p}{\rho} + \frac{V^2}{2} + gz = C;$

p: pressure, v: velocity, z: evelation, γ: the specific weight of the fluid, ρ: the fluid density, g: is the acceleration due to gravity, c: constant.

BE, incompressible: $\frac{p}{\gamma} + \frac{V^2}{2g} + z = C$ or $p + \frac{1}{2}\rho V^2 + \gamma z = C$

p: pressure, v: velocity, z: evelation, γ: the specific weight of the fluid, ρ:the fluid density, g: is the acceleration due to gravity, c: constant.

BE, incompressible, stagnation, $z_1 = z_2$: $\frac{p_2 - p_1}{p_1} = \frac{kMa_1}{2}$

Ma_1: Mach number

$$Ma_1 = \frac{V_1}{c_1}$$

c_1: the speed of sound in the fluid at Point, given by:

$$c_1 = \sqrt{kRT_1}$$

k: the specific heat ratio, T: temperature, R: gas constant

BE, isentropic: $\left(\frac{k}{k-1}\right)\frac{p_1}{\rho_1} + \frac{1}{2}V_1^2 + gz_1 = \left(\frac{k}{k-1}\right)\frac{p_2}{\rho_2} + \frac{1}{2}V_2^2 + gz_2$

BE, isentropic, stagnation: $\frac{p_2-p_1}{p_1}$=$\left[\left(1 + \frac{k-1}{2}Ma_1^2\right)^{\frac{k}{k-1}} - 1\right]$

where $v_1 v_2$and are the flow velocities of the superimposed (inward/outward) flows at locations

where the pressures are $p_1 p_2$and, respectively.

Ma_1: Mach number, k: the specific heat ratio,

Results and Disscusions

Bernoulli did not consider the flow in a dynamic state but only made a law based on the static state, which led to a fundamental error in the nature of physics

Bernoulli's law is not a new problem. Its introduction has been applied and solved several problems. Bernoulli's law is still correct in the case $m_1 = m_2$ (or the Bernoulli equation does not consider the mass factor). For example, the returned results are correct when measuring aircraft speed using a Pitot-static tube.

In the consequences mentioned above, due to the participation of experimental constants (k, R, Ma), the errors of the law have been corrected, leading to the Bernoulli equation becoming an approximate equation. Correctly understanding the Bernoulli equation will significantly reduce the amount of experiments needed in practice, leading to cost and time savings. At the same time, the results of this study will lead to a series of other studies to determine experimental coefficients.

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