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Game Theory and Its Relationship with the Company: A Precious Ally for Business Success, Marketing Activity and Business Risk Prevention

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Abstract
Game theory was developed in the 1950s by many scholars. It was explicitly applied to natural evolution in the 1970s, although similar developments date back to at least the 1930s. It has applications in various fields of the social sciences, as well as in logic, systems theory, and computer science. In recent years, differential games have found several applications in marketing, as shown by international studies. The paper will examine how game theory can be a valid ally both for the marketing activity and a support for the control and containment of the business risk inherent in marketing.

Keywords: Theory games, business risk, marketing, business success

1. Introduction

This paper stems from the desire to demonstrate how mathematics, which many consider a discipline in itself, can become an aid in corporate life. In particular, in the application model that we will illustrate in the following pages, we will highlight (demonstrate) how the theory of differential games can become a valid application tool of considerable importance for controlling the business risk inherent in every business and marketing activity. The model was designed to be applied to one of the marketing mix levers: the product (both product innovation and process innovation). The paper will be divided into two parts; in the first part, in order to guarantee the reader a full knowledge of the subject under investigation, the model on which the construct of differential games is supported will be outlined. In the second part of the paper, however, (we will focus our attention) we will highlight the application of game theory to marketing. The model will be applied to the product and we will try to demonstrate how companies can innovate both the product and the process simultaneously, or innovate the two activities individually; obviously this depends on the initial conditions of the state variables, that is, it depends on the initial homogeneity levels of the product and on the marginal cost of production. Therefore, we can say that the model certainly finds application in the following situations:

1. When companies activate both types of investment from the start of their business;

2. When companies initially invest only in product differentiation and then, in the next phase, focus on product innovation;

3. When companies initially invest only to reduce the cost of production and then, in the next phase, focus on process or product innovation or carry out both;

The next stage of the paper will be the addition to the model of a variable that identifies the risk inherent in the company choices regarding the product, both when it comes to product and process innovation, and an attempt will be made to verify how, also in this context, the model differential games help the firm in its choices. Finally, in the last part of the paper, we will demonstrate how the proposed model can also become a support for controlling and limiting the business risk inherent in marketing. In other words, both the empirical application of the proposed model and the possibility, with the appropriate modifications, of making it applicable to the other levers of the marketing mix will be verified.

2. The model

Let's consider a differential game of chance in continuous time $t \in [0,\infty]$, where at a given moment the companies decide the level of investment and the quantity of production in relation to technological innovation or product innovation or considering both alternatives. Process innovation is formalized to reduce production costs, while innovation is aimed at renewing the product according to the satisfaction of end consumers. The dynamics associated with product innovation are described by the kinematic equation:

$$\frac{ds(t)}{dt} \equiv s = s(t)\left[-x_i(t) - x_j(t) + \delta\right] \quad [1]$$

where x_i represents the amount of effort, economic and otherwise, made by the companyi at the time t to increase product differentiation through a reduction of s(t)i.e. the degree of product substitutability. In fact, the parameter $s(t)$represents the represents the degree of substitutability between the two products and is included between $[0,1]$. If $s(t)=1$identifies that the products are completely homogeneous. On the contrary, however, if $s(t)=0$, , the products are independent of each other and each firm sets its own price as a monopolist. The parameter $\delta \in [0,1]$, instead, it indicates the percentage of depreciation due to aging of the technology which is a common element in enterprises. If the above situations occur, equation (2) can be rewritten as follows:

$$\frac{s}{s(t)} = -x_i(t). - s_j(t) + \delta \quad [2]$$

It is easy to see that the percentage of the firm's marginal fixed costs become linear in the instant investment and over time this is common to all firms that make the same investments. In formula, all this will be:

$$\frac{c_i}{c(t)} = -k_i(t) - \theta k_j(t) + \eta \quad [3]$$

The instantaneous cost of investment in innovative products and innovative processes is given respectively by $\gamma[x_i(t)]^2$ e $\beta[k_i(t)]^2$. It should be noted that both types of investments are financed through internal funds. The parameters γ and β are parameters that respectively measure the efficiency of the R&D sector in an inverse way both in product innovation and in the innovation of the production process. The instant profit is given by:

$$\pi_i(t) = \left[a - c_i(t) - q_i(t) - s(t)q_j(t)\right]q_i(t) - \gamma\left[x_i(t)\right]^2 - \beta\left[k_i(t)\right]^2 \quad [4]$$

We therefore assume that the company strives to maximize the following stream of profits:

$$\prod_i(t) = \int_0^\infty \pi_i(t)e^{-\rho t}dt \qquad [5]$$

This is done under the following control variables $x_i(t), k_i(t), q_i(t)$ where the latter represents the market variable. Function (5) is subject to both the effect of the discount rate $p > 0$ which is presumed to be continuous and common to all companies that share both equation (1) and the following kinematic equation:

$$\frac{dc_i(t)}{dt} \equiv c_i = c_i(t)\left[-k_i(t) - \theta k_j(t) + \eta\right][6]$$

where $k_i(t)$ indicates the company's effort to reduce the cost of production. The parameter $\theta \in [0,1]$ measures the positive technological spillover that the company receives from the competitor's innovative activity process, while $\eta \in [0,1]$ is the percentage of depreciation, which becomes common and constant over time to all companies operating in the sector. The current value assumed in the corresponding Hamiltonian function is:

$$H_i(t) = e^{-\rho t}\left[\pi_i(t) + \lambda_i(t)s + \lambda_{ii}(t)c_i + \lambda_{ij}(t)c_j\right] \qquad [7]$$

Where:

$\lambda_i(t) = \mu_i(t)e^{\rho t}$,

$\lambda_{ii}(t) = \mu_{ii}(t)e^{\rho t}$

$\lambda_{ij}(t) = \mu_{ij}(t)e^{\rho t}$, $\mu_i, \mu_{ii}(t)$ e $\mu_{ij}(t)$ whose associated co-state variables are respectively

$s(t), c_i(t)$ e $c_j(t)$.

Furthermore, where:

$\pi_i(t) =$ represents the instant profit

c_i = represents the constant marginal cost of the firm (i)

s = represents the represents the degree of substitutability between the two products

3. The differential games applied to the product and the risk associated with it

Firms maximize the flow of profits by controlling the following variables $x_i(t), k_i(t), q_i(t)$ and the function described above is subjected to the action of equations (1) and (6) previously exposed. The corresponding Hamiltonian function can be written as:

$$\begin{aligned} H_i(t) = e^{-\rho t}\{&[a - c(t)]q_i(t) - q_i(t)^2 - s(t)q_i(t)q_j(t) - \gamma[x_i(t)]^2 - \beta[k_i(t)]^2 + \\ &+ \lambda_i(t)s(t)[\delta - x_i(t) - x_j(t)] + \lambda_{ii}(t)c_i(t)[\eta - k_i(t) - \theta k_j(t)] + \\ &+ \lambda_{ij}(t)c_j(t)[\eta - k_j(t) - \theta k_i(t)]\} \end{aligned} \qquad [8]$$

We assume $\eta = \delta$ in other words, suppose that the depreciation percentages associated with the technological innovation process and product innovation are identical. Furthermore, we assume that the company has the following initial condition: $s(0) = s_0 \in [0,1]$ and we also consider that companies i, j play simultaneously. Then, the ordinary conditions of control will be:

$$\frac{\partial H_i(t)}{\partial q_i(t)} = a - c - 2q_i(t) - s(t)q_j(t) = 0 \Rightarrow q_i^*(t) = \frac{a - c - s(t)q_j(t)}{2} \qquad [9]$$

$$\frac{\partial H_i(t)}{\partial x_i(t)} = -2\gamma x_i(t) - \lambda_i(t)s(t) = 0 \Rightarrow x_i^*(t) = \frac{s(t)\lambda_i(t)}{2\lambda} \qquad [10]$$

We note that both (9) and (10) contain the state variable $s(t)$ which, therefore, is a common and characterizing element for both companies: that is, the company i and the company j. As a consequence, the open-loop and closed-loop solutions do not coincide. The concept of solution that we adopt is that of the closed-loop of the Nash equilibrium. At this point we consider the reaction between the strategy of the company (player) i and the variable strategy adopted by the company (player) j . This will lead to a balance characterized by perfection in the subgame. It should be noted that the co-state equation adopted by the company i and the company j contains the following reaction effects:

$$-\frac{\partial H_i(t)}{\partial s(t)} - \frac{\partial H(t)}{\partial q_j(t)}\frac{\partial q_j^*(t)}{\partial s(t)} - \frac{\partial H_i(t)}{\partial x_j(t)}\frac{\partial x_j^*}{\partial s(t)} = \lambda_i - \rho\lambda_i(t) \Rightarrow \qquad [11]$$

$$\lambda_i = q_i(t)q_j(t) - \frac{s(t)[q_i(t)]^2}{2} - \frac{\lambda_i(t)\lambda_j(t)s(t)}{2\gamma} + \lambda_i(t)[-\delta + x_i(t) + x_j(t) + \rho]$$

with the following transferability condition:

$$\lim_{t\to\infty} u_i(t)s(t) = 0 \qquad [12]$$

Therefore: $q_i(t) = q_j(t) = q(t)$ e $x_i(t) = x_j(t) = x(t)$. This assumption of symmetry does not involve any loss of generality on the part of the company as long as a Nash equilibrium is adopted as the solution concept. In light of the above, the consequence of the dynamic equation will be:

$$x = \frac{1}{2\gamma}\left[-\lambda s(t) - \lambda(t)s\right] \quad [13]$$

where assume different values depending on the operational context. By imposing the stationarity of state and control we obtain the following equilibrium expressions:

$$s^*(c) = \frac{(a-c)^2 - 2\delta\gamma(\partial + 2\rho) - (a-c)\Psi}{(a-c)^2 + \delta\gamma(\delta + 2\rho)} \quad [14]$$

$$c^*(s) = \frac{1}{2}\left\{a - \frac{\sqrt{(1+\theta)\left[a^2(1+\theta) - 8\beta\eta\rho(2+s)\right]}}{1+\theta}\right\} \quad [15]$$

where:

δ = indicates the percentage of depreciation due to aging of the technology;

$\Psi = \sqrt{(a-c)^2 - 8\delta\gamma(\delta + 2\rho)}$

ρ = discount rate

This is valid as long as $s^*(c)$ and $c^*(s)$ belong to $\Re$, then also $s^*(c) \in [0,1]$ and $c^*(s) \in [0,1]$. So, we note that:

$$s^*(c) \in \Re \quad \text{if} \quad \rho < \frac{(a-c)^2 - 8\delta^2\gamma}{16\delta\gamma} \quad [16]$$

$$c^*(s) \in \Re \quad \text{if} \quad \rho < \frac{a^2(1+\beta)}{8\beta\delta\gamma(2+s)} \quad [17]$$

We note that there certainly exists a series of admissible values for the percentage discount that satisfies (17), while it is possible that $(a-c)^2 < 8\delta^2\gamma$. Therefore, (17) is impossible, which implies that there is no real solution to the product innovation problem. In this case, the companies proceed exclusively to take action for process innovation, while product differentiation remains only the initial condition that we indicate with s_0. This leads to the following Lemma:

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Lemma 1: Companies can be incentivized, depending on strategic choices and operational contexts, to direct their R&D towards process and product innovation. Indeed:

- If

$$\frac{a^2(1+\beta)}{8\beta\delta\gamma(2+s)} > \frac{(a-c)^2 - 8\delta^2\gamma}{16\delta\gamma} > 0$$

then:

i) $\forall\, \rho \in \left[0, \frac{(a-c)^2 - 8\delta^2\gamma}{16\delta\gamma}\right]$ in this case the companies activate process and product innovation;

ii) $\forall\, \rho \in \left[\frac{(a-c)^2 - 8\delta^2\gamma}{16\delta\gamma}, \frac{a^2(1+\beta)}{8\beta\delta\gamma(2+s)}\right]$ in this case the companies invest exclusively in process innovation

- If

$$\frac{(a-c)^2 - 8\delta^2\gamma}{16\delta\gamma} > \frac{a^2(1+\beta)}{8\beta\delta\gamma(2+s)} > 0$$

then

i) $\forall\, \rho \in \left[0, \frac{a^2(1+\beta)}{8\beta\delta\gamma(2+s)}\right]$ in this case the companies activate process and product innovation;

ii) $\forall\, \rho \in \left[, \frac{a^2(1+\beta)}{8\beta\delta\gamma(2+s)}, \frac{(a-c)^2 - 8\delta^2\gamma}{16\delta\gamma}\right]$ in this case, companies invest exclusively in product innovation

- If

$$\frac{a^2(1+\beta)}{8\beta\delta\gamma(2+s)} > 0 > \frac{(a-c)^2 - 8\delta^2\gamma}{16\delta\gamma}$$

then firms can only activate the R&D process

$$\forall\, \rho \in \left[0, \frac{a^2(1+\beta)}{8\beta\delta\gamma(2+s)}\right]$$

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Heading 1 shows, in general, that the business incentive to reduce investment costs is relatively higher than the incentive to increase product differentiation. The explanation for this result can be found by highlighting that, while investing in product differentiation requires a strong financial investment as the differentiations of the original product increase, however, marginal costs at the same time decrease minimally and are included in $\theta \in [0,1]$.Now we focus our attention on realities in which it is possible to evaluate the existence of optimal solutions both in technological innovation and in product innovation. We note that both functions [19] and [20] are concave in the state variables, and admissible for the values of the following parameters $\{a, \beta, \gamma, \delta, \theta, \rho\}$which take on values within the rectangular region defined by $c^*(s) \in [0, a]$ e $s^*(c) \in [0,1]$. All of this highlights four relevant status spaces. Then $s^*(c)$it resolves for $s = 0$for each $s > s^*(c)$, we know that $s < 0$. This implies that in this region, firms invest in product innovation. Similarly, $c^*(s)$it resolves for $c = 0$consequently, for each $c > c^*(s)$we have that $c < 0$. In this region, companies invest but with the aim of reducing the costs of R&D activities and therefore try to reduce the costs of innovation. These two regions cover an equilibrium point within the conical shape of the following figure. Therefore, if the initial condition (c_0, s_0)identify a point that belongs to the cone, companies project their investment choices on both process and product innovation. If, on the other hand, the initial conditions (c_0, s_0)they identify the north-west point of the equilibrium which in the figure is identified as point "A", the firms initially invest in the differentiation of the product.

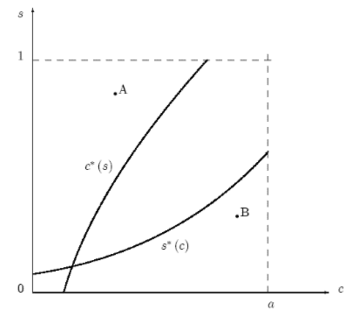
Firms can achieve an optimal situation by choosing the various trajectories present within the cone and which are highlighted in the phase diagram of Figure 17 from which the plausible alternatives are deduced, which are:

1. focus exclusively on product innovation, ensuring that the depreciation of old products and therefore the decrease in profits which is, however, mitigated by a decrease in marginal costs,

2. or focus on product and production process innovation.

The argument set out above finds its raison d'etre, if the initial conditions identify point B as the optimal point.

In the region remaining southwest of the equilibrium point, firms do not invest in both types of innovation, as the percentage of risk of depreciation of innovative products and processes becomes high. The following figure highlights the phase diagram for process and product innovation. In the region remaining southwest of the equilibrium point, firms do not invest in both types of innovation, as the percentage of risk of depreciation of innovative products and processes becomes high. The following figure highlights the phase diagram for process innovation and product.



This discussion implies that the present model offers a theoretical structure from which it can be deduced that companies are projected first towards product innovation and only in a second phase towards process innovation. The theoretical results achieved here also leave room for other equilibrium solutions where process innovation occurs before product innovation, or even the two types of activities coexist within the product life cycle. This type of approach is supported by the studies conducted by Adner and Levinthal which show that there is not necessarily a well-defined hierarchy between the two types of innovation. At this point we can highlight the stability properties of our model for both types of innovation. In this case, the Jacobian matrix will take the form 4 × 4, and it will not be possible to draw the phase diagram. Therefore, in the light of the above, we will have the following matrix:

$$J^* = \begin{bmatrix} \frac{\partial s}{\partial s} = \partial - 2x & \frac{\partial s}{\partial x} - 2s & \frac{\partial s}{\partial c} = 0 & \frac{\partial s}{\partial k} = 0 \\ \frac{\partial x}{\partial x} = \rho + 2x & \frac{\partial x}{\partial s} = \frac{(a-c)^2(3s-2)}{2\gamma(2+s)^3} & \frac{\partial s}{\partial c} = \frac{(a-c)(2-s)s}{2\gamma(2+s)^2} & \frac{\partial x}{\partial k} = 0 \\ \frac{\partial c}{\partial c} = \eta - k(1+\theta) & \frac{\partial c}{\partial s} = 0 & \frac{\partial c}{\partial x} = 0 & \frac{\partial c}{\partial k} = -c(1+\theta) \\ \frac{\partial k}{\partial s} = \frac{c(a-c)}{2\beta(2+s)^2} & \frac{\partial k}{\partial x} = 0 & \frac{\partial k}{\partial c} = -\frac{a-2c}{2\beta(2+s)} & \frac{\partial k}{\partial k} = 0 \end{bmatrix} \quad [18]$$

The Jacobian matrix is made up of four equations that give rise to four eigenvalues. Together, the signal coming from these eigenvalues does not offer an estimate of the value in a simple way, as the reference expression becomes very large and difficult to develop except by means of electronic computers.Furthermore, we cannot obtain the explicit solutions of a function with parameters only for c^* and s^* as if it were two anomalous equations as they are equations whose degree is higher than four. They are obtained by solving the following functions:

$$c - c^*(s) = 0 \quad [19]$$

$$s - s^*(c) = 0$$

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However, we can resort to numerical calculations that allow us to obtain solutions. To solve our problem, we use the following solutions:

$$x^* = \frac{\delta}{2}\ k^* = \frac{\delta}{1+\theta} \qquad [20]$$

and the numerical values for the reference variables will be:

$$a = 1;\ \beta = \gamma = \frac{1}{2};\ \theta = \frac{1}{5};\ \delta = \rho = \frac{1}{20} \qquad [21]$$

Then, we can solve the [19] using simple mathematical calculations and calculate the eigenvalues of J^* or $\{\xi_1, \xi_2, \xi_3, \xi_4\}$. Observing the values of [21] we have that:

$$c^* \cong 0.0042;\ s^* \cong 0.0076\,[22]$$

$$\xi_1 \cong 0.1288;\ \xi_2 \cong 0.0808;\ \xi_3 \cong -0.0308;\ \xi_4 \cong -0.0288\,[23]$$

Using instead:

$$a = 8;\ \beta = \gamma = \frac{1}{2};\ \theta = \frac{1}{2};\ \delta = \frac{1}{20};\ \rho = \frac{1}{40} \qquad [24]$$

we obtain:

$$c^* \cong 0.002;\ s^* \cong 0.0001 \qquad [25]$$

$$\xi_1 \cong 0.0999;\ \xi_2 \cong 0.0499;\ \xi_3 \cong -0.0249;\ \xi_4 \cong -0.0251 \qquad [26]$$

In general, by repeating the same exercise for each admissible parameter, it can be verified that the consequence will be that:

$$\xi_1 + \zeta_2 > 0 \text{ and } \xi_3 + \zeta_4 < 0$$

Therefore, balance is a saddle point. Let's add another variable to our model, namely: $\pm\Phi = (\varepsilon + \varphi) - \Theta$, this represents the risk factor that the company runs by modifying or innovating its production. Φ it is a function of variables $\varepsilon, \varphi, \Theta$; let's see them in detail:

- ✓ $\varepsilon = \frac{1}{N}\sum_{i=1}^{N} a_{1,} + a_2 + \ldots a_n$

where $\varepsilon \in [0,1]$ and i $a_i \in [0,1]$, with that parameter $\varepsilon \in [0,1]$ we estimate the consumer's propensity for an innovative product. If it shows a high propensity to buy on the part of the final consumer. With $\varepsilon = 0$ highlights a low propensity to buy on the part of the final consumer. This parameter is a function of $a_1, a_2, ..., a_n$ which identify the various elements that can hypothetically lead the potential customer to purchase or not a specific product and this materializes in a risk of loss of profit. For example we identify with the ability of the product to satisfy a need. We know that $a_1 \in [0,1]$, therefore if $a_1 = 0$ points out that the customer considers the product's ability to satisfy a need to be null and void. If instead $a_1 = 1$ points out that the customer considers the ability of the product to satisfy a need to be excellent and therefore will certainly choose the purchase of the product.

- ✓ $\varphi = \frac{1}{N} \sum_{i=1}^{N} \aleph_i (S_i)$

with this parameter we estimate the behavior of rival companies against a product and process innovation proposed by the opposing company. Precisely with the parameter $\aleph \in [0,1]$ we estimate the ability of competing companies to adapt to both product and process innovations implemented by the antagonist company. Self $\aleph = 0$ identifies poor company ability, on the part of the competitor, to adapt to innovations. $\aleph = 1$ it identifies, on the other hand, an excellent business capacity on the part of the competing company, to adapt to innovations. But the parameter $\aleph$ becomes a multiplier of S . $S \in [0,1]$ and identifies the intrinsic ability of the rival company to attract customers as well as to keep the customers already loyal customers.If $S = 0$ identifies the company's poor ability to attract customers, if instead $S = 1$ instead, it identifies an excellent corporate ability to attract customers. It should be noted that certainly the company can, thanks to its intrinsic ability, attract customers but must also respond to the needs of the same in order to benefit from this intrinsic added value. In fact, if other companies succeed thanks to new products or by proposing an existing product but with a new variant, then this company added value is canceled. In fact, for example, if $\aleph = 0$,while $S = 1$ at that time $\varphi = 0$ .The two variables highlighted above depend on the estimate that the innovating company makes upstream of its decisions and are supported by a series of market surveys, the study of past events, etc. Therefore, there is a risk of an incorrect assessment. To deal with this risk, we have introduced a corrective in the initial function, namely: $\Theta \in [0,1]$, where with $\Theta = 0$ highlights no type of evaluation error, while with $\Theta = 1$ highlights the maximum error of judgment that can be committed. Let's go back to the matrix [18] and introduce the parameter Φ and in this way we will obtain the Jacobian matrix which will take the form 4 × 4, and it will not be possible to draw its phase diagram. We will therefore have the following matrix:

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$$J^* = \begin{bmatrix} \frac{\partial s}{\partial s} = \partial - 2x \pm \Phi & \frac{\partial s}{\partial x} - 2s \pm \Phi & \frac{\partial s}{\partial c} = \pm\Phi & \frac{\partial s}{\partial k} = \pm\Phi & \\ \frac{\partial x}{\partial x} = \rho + 2x \pm \Phi & \frac{\partial x}{\partial s} = \frac{(a-c)^2(3s-2)}{2\gamma(2+s)^3} \pm \Phi & \frac{\partial s}{\partial c} = \frac{(a-c)(2-s)s}{2\gamma(2+s)^2} \pm \Phi & \frac{\partial x}{\partial k} = \pm\Phi \\ \frac{\partial c}{\partial c} = \eta - k(1+\theta) \pm \Phi & \frac{\partial c}{\partial s} = \pm\Phi & \frac{\partial c}{\partial x} = \times\Phi & \frac{\partial c}{\partial k} = -c(1+\theta) \pm \Phi \\ \frac{\partial k}{\partial s} = \frac{c(a-c)}{2\beta(2+s)^2} \pm \Phi & \frac{\partial k}{\partial x} = \pm\Phi & \frac{\partial k}{\partial c} = -\frac{a-2c}{2\beta(2+s)} \pm \Phi & \frac{\partial k}{\partial k} = \pm\Phi \end{bmatrix} \text{[27]}$$

The four equations of the matrix highlighted above give rise to four eigenvalues, which also in this case, as seen above, it becomes difficult and impracticable to analytically estimate the signal coming from these eigenvalues. To calculate c^* and s^* we resort to [19].At this point to solve the equations that will be obtained from the matrix J^* we can resort to simple numerical calculations that allow us to obtain solutions. To solve our problem, we use the following solutions:

$$x^* = \frac{\delta}{2}\ k^* = \frac{\delta}{1+\theta} \pm \Phi = (\varepsilon + \varphi) - \Theta, \qquad [28]$$

and the numerical values for the reference variables will be:

$$a = 1;\ \beta = \gamma = \frac{1}{2};\ \theta = \frac{1}{5};\ \delta = \rho = \frac{1}{20}; \varepsilon = 0{,}3; \varphi = 0{,}4; \Theta = 0{,}4 \ [29]$$

Then, we can solve the [28] through simple mathematical calculations and calculate the cars of J^* or $\{\xi_1, \xi_2, \xi_3, \xi_4\}$. Observing the values of the[29] We have that:

$$c^* \cong 0.0049;\ s^* \cong 0.0083 \qquad [30]$$

$$\xi_1 \cong 0.1295;\ \xi_2 \cong 0.0815;\ \xi_3 \cong -0.0315;\ \xi_4 \cong -0.0295 [31]$$

In general, by repeating the same exercise for each admissible parameter, it can be verified that the consequence will be that:

$$\xi_1 + \zeta_2 > 0 \ \text{ e } \ \xi_3 + \zeta_4 < 0$$

Therefore, even in this case, this balance is a saddle point[1]. Comparing the [30] and [33] with the [25] and [26] differences in values are denoted precisely due to the value of the variable Θ. The equilibrium generated by the theoretical arrangement that we have analyzed is in line with the conclusions outlined by Adner and Levinthal, indicating that the life cycle of the product and

[1]IIn mathematical analysis, a saddle point of a real function of several real variables $f : \Re^n \to R$ it is a critical point*P*of the domain of the *f*in which the Hessian matrix is indefinite: that is, it is neither a positive semidefinite matrix, nor a negative semidefinite matrix.

technology cannot necessarily follow the ritual first of product innovation and then of technological innovation, model, on the other hand, outlined by conventional wisdom.

4. Conclusion

The paper was divided into various parts, wanting to give the right importance to every single topic dealt with and at the same time disseminate those elements that would then, in the end with the presentation of the model, allow us to draw that common thread that united the arguments developed, demonstrating that the various disciplines can cooperate with each other, creating added value. Starting from this initial assumption, supported among other things by international literature, the entire structure of the work was conceived. In the application model illustrated in the previous pages we have shown how the Theory of Differential Games can become a valid application tool of considerable importance not only in the definition of strategic operational choices inherent in the marketing activity but at the same time it can become a support to control and stem the business risk inherent in every business activity and also in marketing. Using the proposed model, we have succeeded in connecting marketing, business risk and game theory. In particular, the model has been applied to the product and in this case we have shown how companies can simultaneously activate both product and process innovation, or activate the two activities individually and this depends on the initial conditions of the state variables, i.e. from the initial levels of homogeneity of the product and from the marginal cost of production. Therefore, we can say that the model certainly finds application in the following situations:

1. When companies activate both types of investment from the start of their business;

2. When companies initially invest only in product differentiation and then, in the next phase, orient themselves towards product innovation;

3. When companies initially invest only to reduce the cost of production and then, in the next phase, focus on process or product innovation or carry out both;

The equilibrium generated by the theoretical arrangement that we have analyzed is in line with the conclusions outlined by Adner and Levinthal indicating that the life cycle of the product and technology cannot necessarily follow the ritual first of product innovation and then of technological innovation, model, on the other hand, outlined by conventional wisdom. The next phase of the work was the addition to the model of a variable that identifies the risk inherent in the company choices regarding the product, both when it comes to product and process innovation, and it has been verified that also in this context the model differential games help the firm in its choices. In other works, both the empirical application of the proposed model and the possibility, with the appropriate modifications, of making it applicable to the other levers of the marketing mix will be verified.

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