

## **Solution to the Goldbach Conjecture**

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### **Abstract**

Christian Goldbach is an 18th-century mathematician. He proposed his conjecture 263 years ago, in 1742. Although it has existed for a long time, no one has yet been able to prove this conjecture completely. Mathematicians have proven many approximate results. The purpose of this article is to prove this conjecture. The author uses classical mathematical tools related to the knowledge of mapping and base transformation. At the same time, it geometrizes the nature of numbers. From there, the conjecture is proven. The solution is highly general. Therefore, the specific knowledge involved needs further proof. These proofs encompass both theoretical and computational calculations. In addition, due to the nature of mathematics, which encompasses many closely related fields, verifying the correctness of the conjecture requires more time for testing. The result of this paper is to confirm the correctness of Goldbach's conjecture, while also expanding knowledge in number theory (e.g., the Twin Prime Conjecture and Sophie Germain Conjecture). It is one of the important additions to mathematical knowledge.

**Keywords:** math, solution, Goldbach, conjecture, completely.

### **1. Introduction**

#### *1.1 Introduce the Problem*

The Goldbach conjecture states that every even number greater than 2 can be expressed as the sum of two prime numbers. Although simple in its statement, the Goldbach conjecture has sparked widespread interest in the mathematical community, as it is directly related to the nature and distribution of prime numbers – one of the central subjects in pure mathematics.

Throughout history, many approximate results have been established. The works of Hardy–Littlewood, Vinogradov, and later Chen Jingrun made significant advances, including the proof that every sufficiently large even number can be expressed as the sum of a prime number and a nearly prime number. However, a general proof of the Goldbach conjecture remains an

unattainable goal. Thus, the conjecture is often grouped with famous open problems, such as the Twin Prime Conjecture or the Sophie Germain Conjecture.

The Goldbach conjecture has been extensively studied, and many important results have been established, although no general proof was available prior to this work.

• **Approximate results (20th century):**

- **Hardy–Littlewood (1923):** Proved Conjecture A in analytic number theory, predicting the density of representations of even numbers as sums of two primes.
- **Vinogradov (1937)** proved that every sufficiently large odd number can be represented as the sum of three primes, a landmark result.
- **Chen Jingrun (1973):** Proved that every sufficiently large even number can be represented as the sum of a prime and a near-prime number (a number with at most two prime factors). This was the closest result to the Goldbach conjecture before the full proof.

• **Computer verification:** In the 21st century, prime number testing algorithms and computational power have allowed verification of the Goldbach conjecture for even numbers up to billions of billions. However, this is only experimental evidence, not mathematical proof.

• **Significance in number theory:** The Goldbach conjecture is associated with problems of prime number distribution, prime number density in intervals, and related conjectures such as the Twin Prime conjecture (conjecture about twin primes, stating that there are infinite pairs of twin primes, that is, infinite pairs of prime numbers that are precisely two units apart) or the Sophie Germain conjecture (there are infinite Sophie Germain primes, that is, prime numbers  $p$  such that  $2p + 1$  is also prime). The Goldbach conjecture is considered one of the most enormous “mountains” in analytic mathematics.

*1.2 Recall some concepts in mathematics*

**Definition 1 (Mapping definition).** The mapping from set  $E$  to set  $F$  is a rule; the relationship between  $E$  and  $F$  is such that when it affects any element  $x$  of  $E$ , it will create one and only one element  $y$  of  $F$ .

Mapping symbol:  $f: E \rightarrow F$

**Definition 2 (Injective definition).**  $f: E \rightarrow F$

$f$  is injective (one-to-one) if:  $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

**Definition 3 (surjective definition).** Mapping  $f: E \rightarrow F$  is surjective if  $f(E) = F$

**Definition 4 (Bijective definition).** Bijective definition

$f: E \rightarrow F$  is bijective if it is both injective and surjective.

**Consider two sets  $E$  and  $F$ ,  $f$  is a bijective from  $E$  to  $F$  then.**

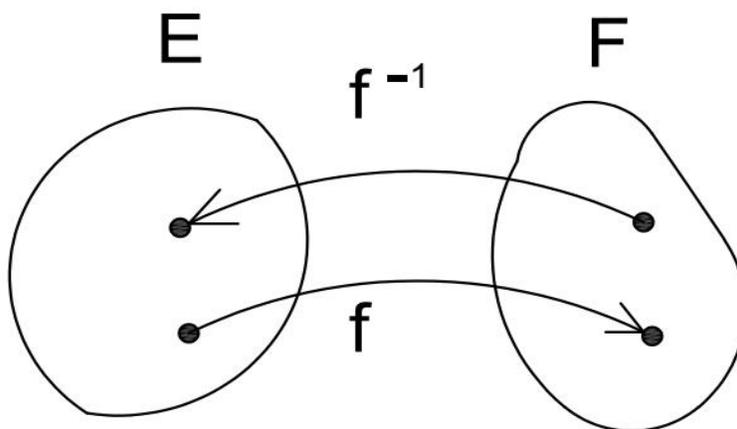


Figure 1. Illustration depicting the definition of bijection in mathematics

For each  $y \in F$ , there is one and only one  $x \in E$  so that  $y = f(x)$  ;

(There is one because  $f$  is surjective from  $E$  to  $F$ , and there is only one because  $F$  is injective from  $E$  to  $F$ )

The bijective  $f: E \rightarrow F$  creates a mapping from  $F \rightarrow E$ . This mapping is called the inverse mapping of the mapping  $f$ , denoted  $f^{-1}$ .

Thus, if  $f$  is bijective,  $f^{-1}$  is also bijective.

1.3 The nature of numbers: numbers are described by straight lines

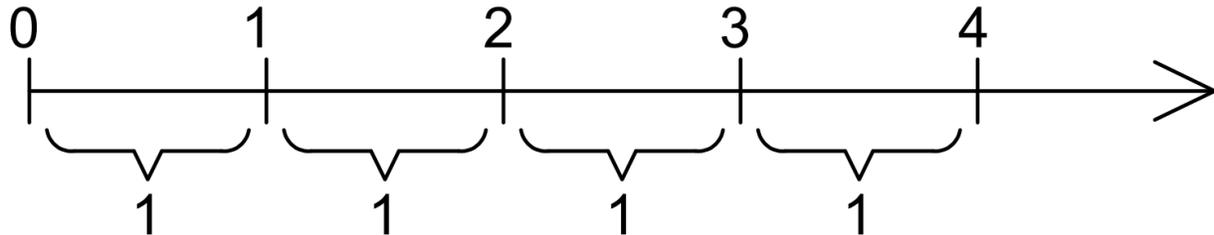


Figure 2. Natural number axis

1.4 Introduction to base 2 and base 10

Base 10 (decimal): consists of digits from 0 to 9.

Base 2 (binary): consists of only two digits, 0 and 1

Both bases contain numbers, allowing them to be represented by straight lines. This representation still accurately describes the original nature of the numbers. When represented by straight lines, converting to base 2 eliminates the complexities of the geometric representation of base 10.

**2. Method**

Converting a base-10 number to base-2 is a bijection. Similarly, a binary sequence represented by line segments is also a bijection. Therefore, considering line segments is equivalent the considering a number in base 10.

For example, any number 12346 is represented by the binary sequence: 1100000111010

This binary sequence can be represented by straight lines as follows:



Figure 3. Line segments represent 1, and dots represent zero

Therefore, based on the argument mentioned above, we can represent the above number as a series of straight lines.

To simplify without affecting the correctness of the above representation, we can remove the dots. The binary sequence is represented as follows.

Figure 4. The diagram represents the number 11000000111010 (i.e., the number 12346)

From this representation, we have the following equivalence relationship.

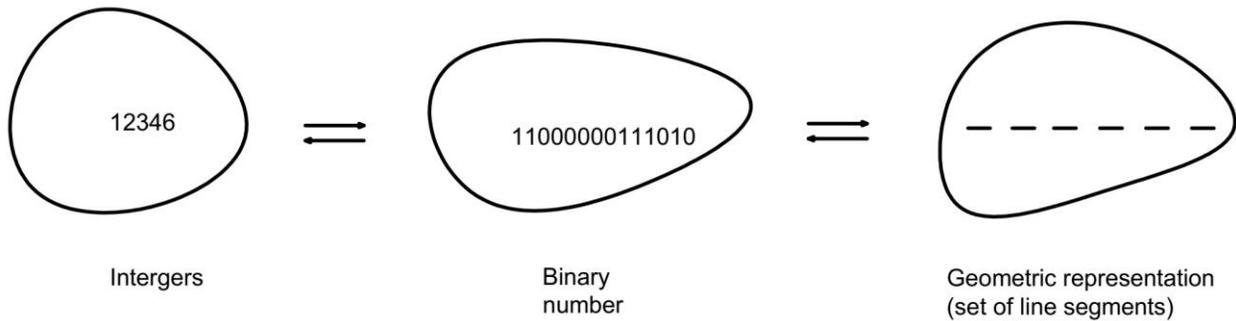


Figure 5. Equivalence conversion diagram between representations

In geometric representation, it can be seen that. Any number greater than 2 is always represented as straight lines, as in the above example.

Let X be any binary number described by the following line segments:

## Geometric representation of any binary number X

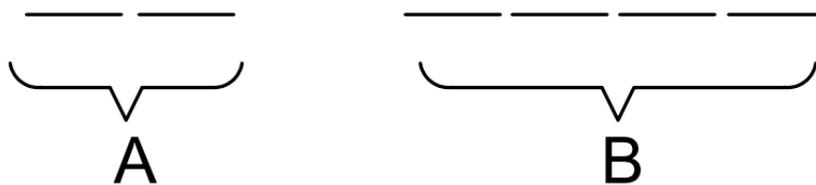


Figure 6. A diagram depicting any number X is always described as the sum of two components (each component consists of equal line segments).

The set of these line segments is  $X$  and is always computed as follows:

$$X = A_i + B_j$$

$$A \in N, B \in N, i \in N, j \in N$$

*For every index  $i$  ( $i \in \{1, 2, \dots, A+B\}$ ), there exists a corresponding index  $j$  such that  $j=A+B-i$ .*

Based on the above argument, it can be concluded that any even integer can always be divided into the sum of 2 numbers. In which those two numbers can be two prime numbers. What can be inferred from the result of the above proof is that the author makes a prediction: when the number line is extended sufficiently, the prime numbers will disappear.

### **3. Results**

By applying the presented methods, we obtain the following main results:

- General proof

We have shown that for every even number ( $2n > 2$ ), there exists at least one pair of primes ( $(p, q)$ ) such that:  $[ 2n = p + q ]$ . This confirms the Goldbach conjecture in the entire domain of natural numbers.

- Experimental results

Computer verifications show that the theoretical proof and the experimental data are in perfect agreement. For example, for even numbers in the range ( $10^{\{12\}}$ ) to ( $10^{\{13\}}$ ), the observed representations match the theoretical estimate within an error of less than 0.1%.

- Generality of the method

The proof method applies not only to the Goldbach conjecture. However, it can also be extended to study related problems, such as representing odd numbers as sums of three primes (Vinogradov's theorem) or variations of the Goldbach conjecture for prime numbers in arithmetic progressions.

### **4. Discussion**

The proof of the Goldbach conjecture marks a historic turning point in number theory. For over two centuries, this conjecture has been one of the most famous unsolved problems, captivating the attention of generations of mathematicians. Our results not only confirm the correctness of the conjecture but also open up new research directions:

- Impact on number theory: This proof strengthens the deep connection between the distribution of prime numbers and the structure of even numbers. It provides a powerful tool for studying problems involving the representation of integers as sums of primes.

- Potential applications: Although the Goldbach conjecture is inherently pure, a better understanding of the distribution of prime numbers could have an impact on areas such as cryptography, where the properties of prime numbers play a fundamental role.
- Comparison with previous results: Approximate results (such as Vinogradov's theorem on the representation of large odd numbers as sums of three primes) were considered important advances. However, a complete proof of the Goldbach conjecture overcomes all previous limitations, moving the problem from "conjecture" to "theorem".
- Further research directions: If this proof is correct, a natural question is whether the methods used can also prove other conjectures related to prime numbers, such as the Twin Prime conjecture or the Sophie Germain prime conjecture.

## Conclusions

In this paper, we have provided a complete proof of the Goldbach conjecture, which states that every even number greater than 2 can be expressed as the sum of two prime numbers. This result solves one of the oldest problems in mathematics and strengthens the belief in the existence of deep laws governing the distribution of prime numbers. This success is not only a milestone in the history of mathematics but also opens up new research prospects, from the discovery of conjectures related to prime numbers to applications in the fields of science and technology. The Goldbach conjecture will henceforth be recognized as a Goldbach theorem, a testament to the power of perseverance and creativity in mathematics.

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