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Surface Design of a Toothed Soil-engaging Element for Soil Tillage

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Abstract
A mathematical model has been proposed, which, under certain conditions, will enable the development of a toothed element with a working surface shape that satisfies the agrotechnical, technological, and economic parameters during soil cultivation.

The working element is made as a block of teeth. The tooth recesses and protrusions in the horizontal plane are shaped according to alogarithmic spiral, with the protrusions forming a fourth-degree parabola.

The toothed element equation is presented in the form of a combination of rotation, displacement, and compression matrices.

The working surface is defined kinematically as the trajectory of points of the generating logarithmic spiral.

Equations are derived describing the surface of the working element at the areas of tooth recesses and protrusions.

Keywords: toothed element, soil, tooth recesses and protrusions, logarithmic spiral, fourth-degree parabola, mathematical model, matrix-vector solution.

Introduction

The productivity of Ukraine's chernozem, which serves as the nation's primary economic asset, is directly influenced by the efficiency of cultivation techniques. The level of soil loosening depends on the geometry of the operating tool and the kinematics of its movement in the work process. Existing studies demonstrate that the tool's geometric parameters and kinematic

Vol. 11, No. 02; 2026

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characteristics determine the energy efficiency of soil processing, while also affecting the mechanical stresses exerted on the soil and the equipment. To meet the above requirements for soil treatment, research on the interaction of the operating tool with the soil was carried out. When doing so, the soil can be viewed as a medium with linear elastoviscous deformation. In the event of contact of linear-elastoviscous bodies, the issue of creating a mathematical model of the working body is reduced to solving problems by methods of elasticity theory based on the "correspondence principle" [1, 2].

The task of designing a toothed working element for tillage machines is reduced to selecting a mathematical model that satisfies all the criteria required for the working element in both technological and economic terms, considering the physical and mechanical properties of the soil [2, 3].

The working tool is made as a block of teeth (Fig. 1), which, in the horizontal plane, consist of recesses shaped according to a logarithmic spiral and protrusions profiled as a fourth-degree parabola.

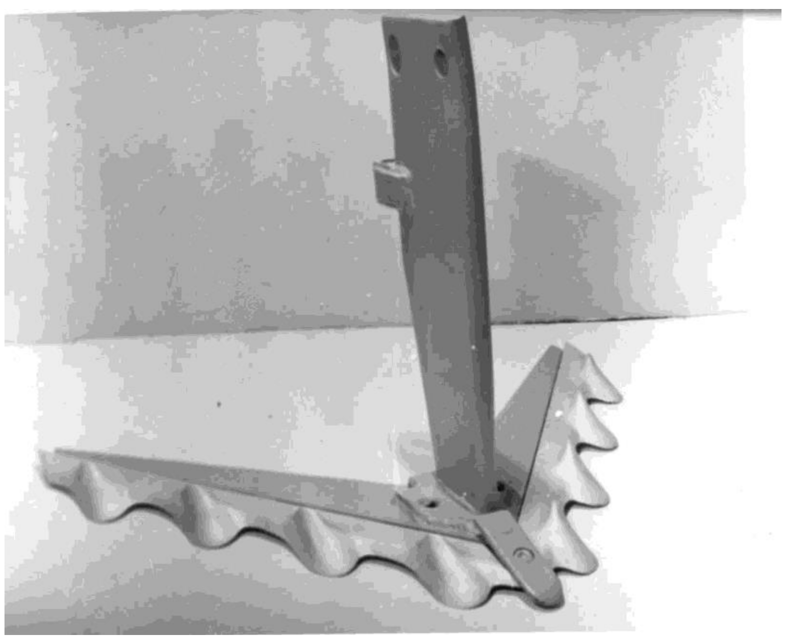

Figure 1 – General view of the toothed working element for tillage.

In horizontally projecting planes, the section of a tooth represents a family of logarithmic spirals:

$$r_{\psi_i} = r_0 e^{\psi tg\phi}, \qquad (1)$$

Vol. 11, No. 02; 2026

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where r_{ψ_i} is the current radius vector;
r_0 is the initial radius vector;
$tg\phi$ is the coefficient of internal friction ;
ψ is the current angle of the radius vector of the spiral.

The equation of the surface of the toothed working element for soil cultivation can generally be represented using a matrix [5]:

$$A_{ij} = \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ \hline 0 & 0 & 0 & 1 \end{array}\right), \quad (2)$$

where $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is a third-order matrix describing rotation in three-dimensional space;

$(a_{14}, \rightleftarrows a_{24,} a_{34})^T$ are components of translational displacement.

The working surface is defined kinematically as the trajectories of points on the generating logarithmic spiral (Fig. 2), located in plane OX_2X_3 and under going rotational and translational motion.

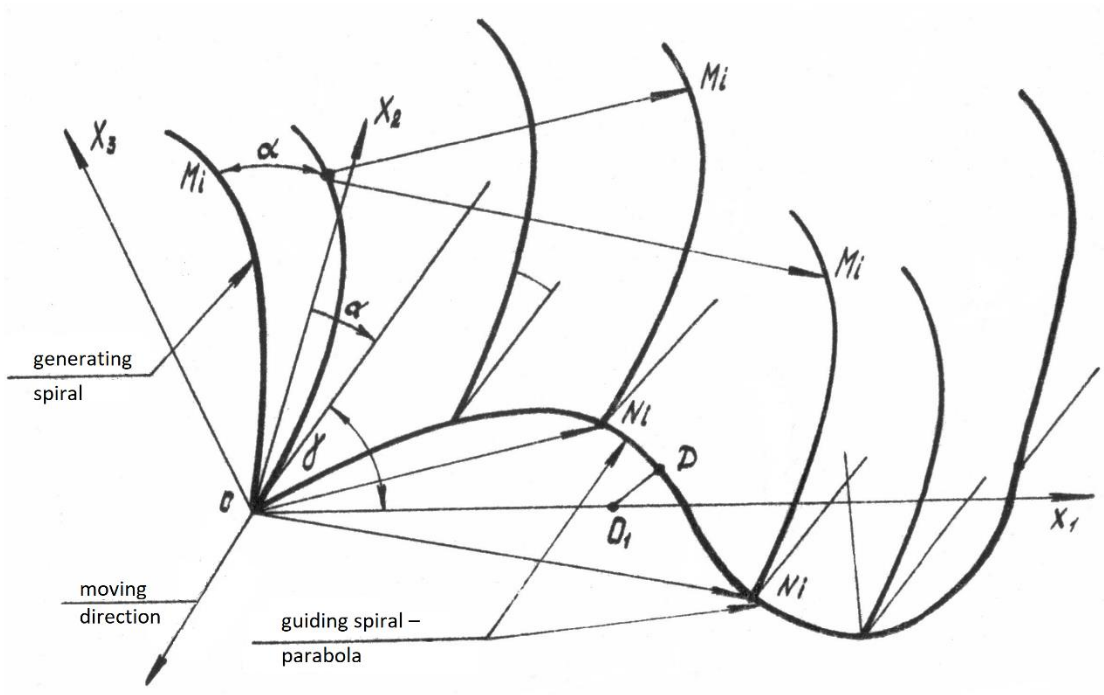

Figure 2 – Kinematic model of the surface of the working element.

The rotation of the points M_i of the spiral around the axis OX_3 at the angle α = 90-γ (where γ is the sweep angle of the tine of the working element) is described by the rotation matrix:

$$A_{вр} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

The coordinates of the current points M_i of the generating logarithmic spiral, determined from equation (1), where $\varphi = 45^0$ and $\psi = -45^0 ... 50^0$ will have the following values (Fig. 3):

$x_1^G = 0,\ x_2^G = r_{\psi_i} \sin\psi_i\ ;\quad x_3^G = r_{\psi_i} \cos\psi_i.$

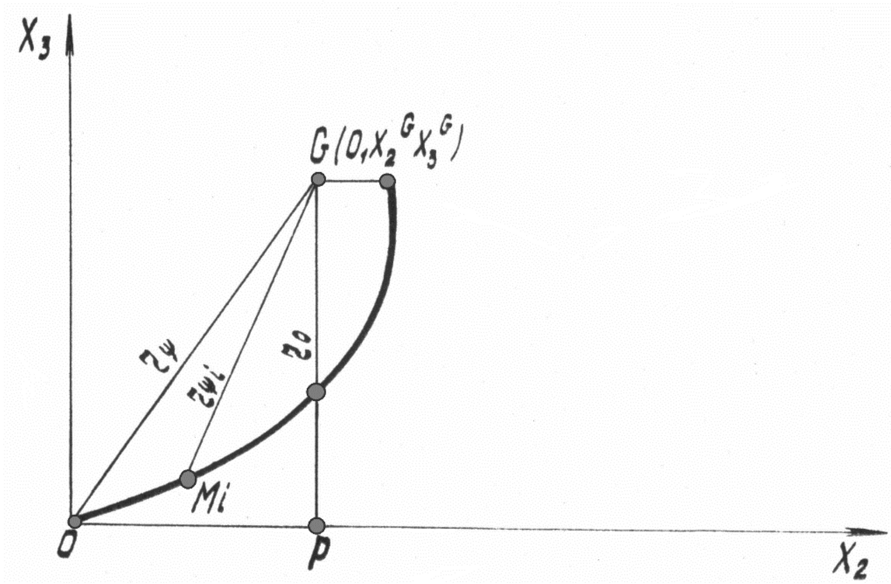

Figure 3 – Type of the generating spiral.

The coordinates of the current points M_i of the generating spiral in Cartesian coordinates, depending on ψ are written as:

$x_1^{r\psi_i} = 0;$

$$x_2^{r\psi_i} = OP - r_{\psi_i} \sin \psi_i \, ; \qquad (4)$$

$x_3^{r\psi_i} = GP - r_{\psi_i} \cos \psi_i .$

The translational displacement of points M_i along the concave or convex section isdescribed by the displacement matrix:

$$A_n = \begin{pmatrix} 1 & 0 & 0 & \Delta x_1^{N_i} \\ 0 & 1 & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (5)$$

where $\Delta x_1^{N_i}$, $\Delta x_2^{N_i}$, $\Delta x_3^{N_i}$ arethe components of the displacement vector.

Thus, the overall displacement of the points of the generating logarithmic spiral is determined by scalar multiplication of the matrices:

$$A = A_n \cdot A_{вр} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & \Delta x_1^{N_i} \\ \sin\alpha & \cos\alpha & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (6)$$

Then, the equation of the surface of the toothed working element can be represented as:

$$Y_i = A \cdot X_i, \quad (7)$$

where X_i are the coordinates of the current point M_i on the generating spiral, determined by equations(4);

Y_i are the coordinates of surface points of the toothed working element, which depend on parameter$\psi_i$on the generating logarithmic spiral, and the displacement vector ON_idetermined by the position of the current point $N_i$on the concave or convex section of the cutting edge.
For the convex section, the coordinates of the current point$N_i$of the displacement vector along the logarithmic spiral (Fig. 4) are determined by the equations:

$$x_1^{N_i} = x_1^{O_1} + r_{\theta_i} e^{\theta_i tg\phi} \cos\theta_i ;$$
$$x_2^{N_i} = r_{\theta_i} e^{\theta_i tg\phi} \sin\bar{\theta}_i ; \quad (8)$$
$$x_3^{N_i} = 0,$$

where θ_i=180° are initial angles;

$\bar{\theta}_i = 20^\circ + \theta_i$ are the coordinates of point$O_1(S/2, 0,0)$;

S is tooth pitch.

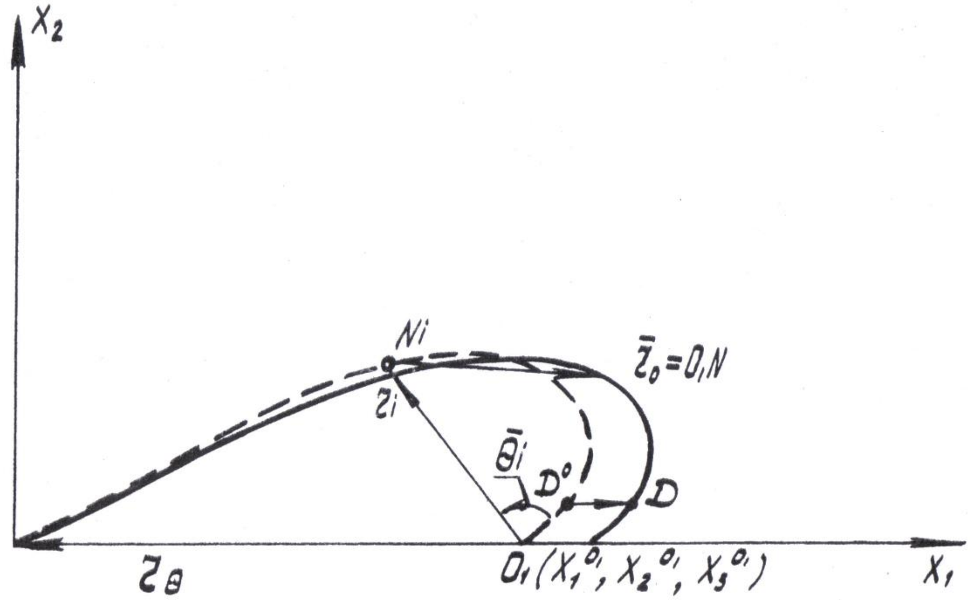

Figure 4 – Recessspiral transformations.

Due to the elongation of the logarithmic spiral of the concave profile along the OX_1 axis, a coefficient is introduced

$$\text{к} = \frac{1}{\cos\alpha},$$

and the elongation transformation is represented by the matrix:

$$A^{y}_{впад} = \begin{pmatrix} \kappa & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (9)$$

Considering equation (9), the coordinates of the points of the translational displacement vector can be determined as:

$$\begin{aligned} &\Delta x_i = A^{y}_{\text{впад}} \cdot x_i; \\ &\Delta x_1^{N_i} = \text{к} x_1^{N_i}; \\ &\Delta x_2^{N_i} = x_2^{N_i}; \\ &\Delta x_3^{N_i} = 0. \end{aligned} \qquad (10)$$

For the protrusion section, the coordinates of the current point N_i (Fig. 5) of the translational displacement vector along the parabola $x_2 = x_1^4$ are determined as the product of the following transformations:

a) compression along the OX_2 axis with coefficient μ, described by the matrix:

$$A_{\text{выст}}^{\text{сж}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad (11)$$

b) elongation along the OX_1 axis with coefficient k, described by matrix (9);
c) translational displacement along the vector CD, described by matrix (5).

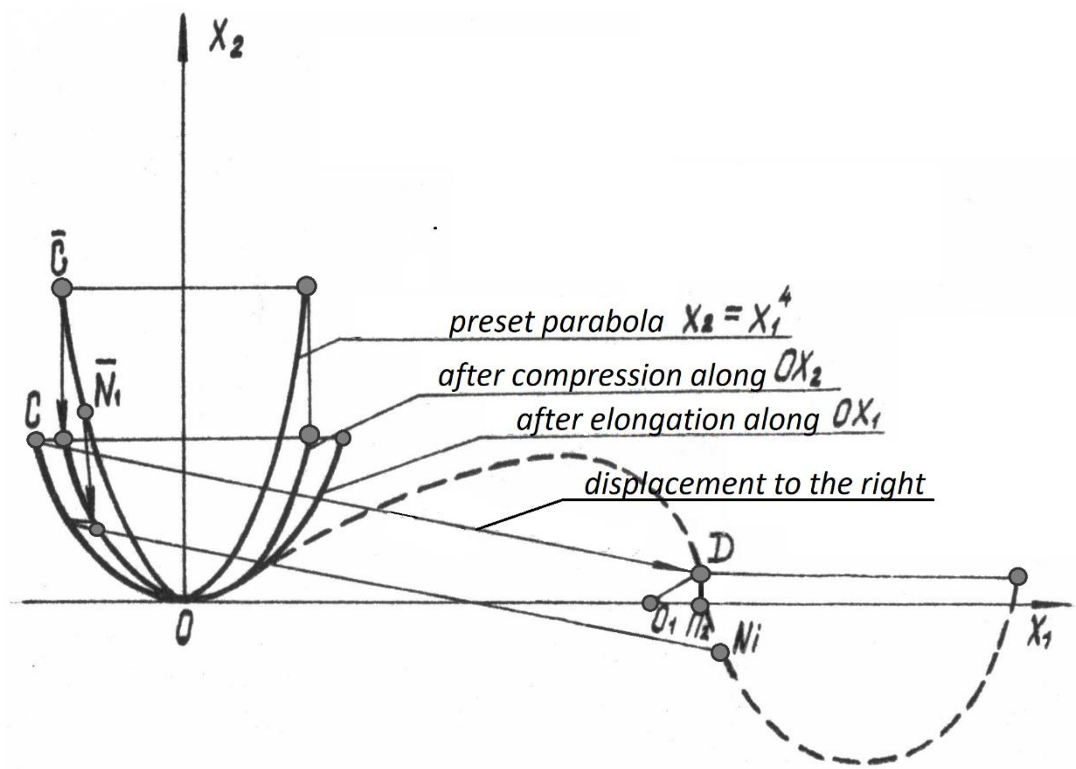

Figure 5 – Coordinates of the current point N_i of the parabola in the section of protrusions.

Then, the overall transformation of the parabola is described by the matrix
$A_{\text{общ.}} = A_n \cdot A_y \cdot A_{\text{сж}}$;

$$A_{общ.} = \begin{pmatrix} k & 0 & 0 & \Delta x_1^{CD} \\ 0 & \mu & 0 & \Delta x_2^{CD} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (12)$$

The coordinates of the current point N_i on the parabola in matrix form are:

$$\Delta x_i = A_{общ.} \cdot X_i, \quad (13)$$

where X_i arethe coordinates of the points of the original parabola $x_2 = x_1^4$;

Δx_i –are the coordinates of the transformed parabola points which are determined as:

$$\Delta x_1 = kx_1 + \Delta x_1^{CD};$$

$$\Delta x_2 = \mu x_2 + \Delta x_2^{CD}; \quad (14)$$

$$\Delta x_3 = 0.$$

The equation of the surface of the toothed working element (7) in matrix form can be represented as:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & \Delta x_1^{N_i} \\ \sin\alpha & \cos\alpha & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1^{M_i} \\ x_2^{M_i} \\ x_3^{M_i} \\ 1 \end{pmatrix}, \quad (15)$$

where $x_1^{M_i}, x_2^{M_i}, x_3^{M_i}$ –are the coordinates of the generating spiralpoints;

$\Delta x_1^{N_i}, \Delta x_2^{N_i}, \Delta x_3^{N_i}$ are the coordinates of the cutting-edge points.

This equation describes the surface of the tooth along the concave (recess) and convex (protrusion) sections.

In the final form, the coordinates of the tooth surface points can be expressed as follows:

For the concave section:

$$y_1 = \sin\alpha\left(OP - r_{\psi_i}\sin\psi_i\right) + k\left({}^{S}/_{2} + r_{\theta_i}\cos\bar{\theta}_i\right);$$

$$y_2 = \cos\alpha\left(GP - r_{\psi_i}\sin\psi_i\right) + r_{\theta_i}\sin\bar{\theta}_i; \quad (16)$$

$$y_3 = GP - r_{\psi_i}\cos\psi_i,$$

For the protrusion section:

$$y_1 = \sin\alpha\left(OP - r_{\psi_i}\sin\psi_i\right) + kx_i + \Delta x_1^{CD};$$

$$y_2 = \cos\alpha\left(GP - r_{\psi_i}\sin\psi_i\right) + \mu x_2 + \Delta x_2^{CD}; \qquad (17)$$
$$y_3 = GP - r_{\psi_i}\cos\psi_i.$$

Conclusion

The obtained mathematical model (16), (17) fordescribing the surface of a toothed working tool allows, under various conditions, taking into account processing depth, agro technical requirements for soil crumbling quality, physical and mechanical soil properties, cutting-edge geometry, and the number of teeth, to generate a family of toothed soil-engaging elements that are operational under specific conditions. The surface shapes of the working tools depend on the values of the functional parametersk, μ, θ, ψ.

References

Baukov A.V., Kushnarev A.S. (1976) Opredeleniye linii skolzheniya v pochve pered deformatorom [Determination of the slip line in the soil in front of the deformer]. *Scientific works of Ukrainian Agricultural Academy*, vol. 151, pp. 21-26

Shtaerman I.L. (1949) *Kontaktnyye zadachi teorii uprugosti* [Contact problems of the theory of elasticity]. Moscow-Leningrad: State Publishing House of Technical and Theoretical Literature.

Efimov A.B. (1966) Osesimmetrichnaya kontaktnaya zadacha dlya lineynykh vyazko-uprugikh tel [Axisymmetric contact problem for linear viscoelastic bodies]. *Moscow State University Bulletin*, no3, pp. 84-89.

Kushnarev A.S. (1981*) Mekhaniko-tekhnologicheskiye osnovy protsessav zaimodeystviya rabochikh organov pochvoobrabatyvayushchikh mashin i orudiy na pochvu* [Mechanical and technological foundations of the process of interaction of operating bodies of tilling machines and tools on the soil]. Doctor of Technical Sciences abstract of the dissertation. Chelyabinsk: Chelyabinsk Institute of Agricultural Mechanization and Electrification.

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